Submitted to the Gravity Research Foundation’s 2006 Essay Contest

How Fundamental is the Curvature of Spacetime?
A Solar System Test

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Abstract

Are some paths and interactions immune to the gravitational curvature of spacetime? The paths of virtual particles might be – the effect is unexplored experimentally. Were a quantum theory of gravity moderated by virtual gravitons that are themselves susceptible to gravitational curvature, to obey the weak equivalence principle, and to follow null geodesics, then gravitational acceleration might be slightly but detectably enhanced by the gravitational lens effect of our Sun at outer solar system locations opposite nearby stars. It is shown that this “virtual equivalence principle” is testable by precisely placing accurate clocks out near the orbit of Neptune.

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Curvature was postulated to be a fundamental attribute of spacetime by Einstein in General Relativity. More directly, the weak equivalence principle states that gravity should accelerate all objects equally independent of their composition. How pervasive is the weak equivalence principle? Given the quantum domain where forces are commonly portrayed as being conveyed by virtual particles, are these virtual particles constrained to follow paths dictated by the weak equivalence principle? If gravity itself can be described by a quantum exchange of particles, do these virtual gravitons follow paths dictated by the weak equivalence principle? The conjecture that virtual particles follow null geodesics, like photons, and hence obey the weak equivalence principle, will hereafter be dubbed the Virtual Equivalence Principle (VEP).

Experimental tests falsifying this aspect of gravity’s prevalent equivalence principle will be described in this essay as becoming practical for the first time. Here, a single realization of the VEP that has become testable will be discussed in some detail. The stated falsifiable hypothesis will be: do virtual gravitons follow the null geodesics of light? It is notoriously difficult to theoretically predict the effects of gravity on itself. Nevertheless, regardless of which gravitational theory one believes, regardless of how one feels a quantum theory of gravity should be cast, real world tests of the VEP can constrain how a quantum theory of gravity can act in a way never before explored.

Previously, in the laboratory, testing this hypothesis was not practical. In short, the relative deflection of real photons in the laboratory, for example passing a 100 kg sphere with the density of iron, is in the realm of \( \alpha = 2R_S/b \sim 10^{-28} \) arcseconds, where \( R_S \) is the Schwarzschild radius of the lens and \( b \) is the impact parameter. This change in direction that is too slight to measure in practice.

Measurability of similar deflections becomes significantly easier in outer parts of our Solar System. There, the massive Sun itself can be used as the deflecting body, and the gravitational lens effect of the Sun becomes significant at its focal length. The minimum focal length for the “opaque Sun”, meaning the distance from the Sun where radiations such as visible light, to which the Sun is opaque, first come to a focus, is about 550 AU [3]. Parallel rays that graze the Sun’s limb meet at 550 AU. Note that 550 AU is only the minimum focal length of the opaque Sun, since parallel rays that pass further out than the Solar limb do come to a focus, but only further out than 550 AU from the Sun.

A less well known focal length of the Sun is the “transparent focal length” that applies
to radiations that can traverse the Sun’s interior. These radiations first come to a focus at about 25 AU [1, 5]. The Sun’s transparent focal length is much shorter than its opaque focal length because the Sun is very much centrally condensed, with much of the mass that causes gravitational lens deflection near its center. At 25 AU, parallel rays that pass near the center of the Sun, actually at about 0.2 solar radii, meet. Parallel rays that pass closer to the Sun’s center do meet but only outside 25 AU. Similarly, parallel rays that pass further to the Sun’s center do also meet, but also only outside 25 AU. Therefore, as above with the opaque Sun, 25 AU is only the *minimum* focal length of the transparent Sun.

What comes into focus at 25 AU? Clearly real radiations able to traverse the dense Sun do. These likely include types and energy bands of neutrinos and gravitational radiation. The specific hypothesis being discussed, however, is that virtual particles, in particular virtual gravitons, would be deflected like light and travel on null geodesics. If so, the curvature of spacetime would be discovered to be fundamental in a way never tested before. Then, virtual gravitons from a mass far opposite the Sun would be focused just like real radiations, starting at 25 AU from the Sun. A small volume of space would be created where gravitational acceleration toward the distant mass would be increased. However, the great distance to this mass and the presence of the intervening Sun would mean that the actual increase in overall acceleration would be quite small.

To compute detectable attributes, the truth of the falsifiable hypothesis of the virtual gravitons obeying the VEP will now be assumed. Then the bounding surface for where slightly enhanced accelerations would exist is the space where the center of the Sun projects into the (unlensed) position of the distant mass. The internal space is dubbed a “gravitational hollow” since no actual mass is present there. Geometrically, inside a “gravitational hollow”, the gravitational acceleration toward a mass far in the distance behind the foreground gravitational lens, in this case the Sun, becomes magnified. The gravitational hollow would be a thin cone extending from about 25 AU out across the universe. The width of a gravitational hollow at 25 AU can be computed to be [6] \( w_{\text{hollow}} = 85 \text{ km}(R_*/R_\odot)(1 \text{ pc}/D_*) \) where \( R \) stands for radius, \( D \) for distance, \( * \) denotes the star being magnified and \( \odot \) stands for our Sun. The slight anomalous acceleration there would be (as derived in [6]): \( (\Delta a/a) = (2 \times 10^{-4})(0.2 R_\odot/R_*)(M_*/M_\odot)(1 \text{ pc}/D_*) \), where \( a \) stands for acceleration in the direction of the distant mass and \( M \) stands for mass. In general, the closer and more massive the mass, the larger and deeper its Sun-created gravitational hollow on the opposite side of
the Sun.

It should be stressed that although gravitational hollows are as numerous as stars surrounding the Sun, most of these hollows are quite thin so that the vast majority of space is outside gravitational hollows and unaffected. It is therefore a rare random object that would encounter a large and deep hollow. Graphically, the system of gravitational hollows that surrounds the Sun might be visualized as the thin spines of a sea-urchin. Different stars near the Sun would create different thin spines extending well away from the Sun.

In practice, near the Sun, the increase in acceleration toward distant bodies is slight and would need the accuracy of modern clocks for detection. As an example, take the closest star system the Alpha Centauri system. The most massive star in that system is Alpha Centauri A, a star very much like our Sun. A recently published mass of Alpha Centauri A is 1.105 $M_\odot$, with a radius of 1.224 $R_\odot$ and a distance of 1.34 pc [4].

A very accurate clock placed inside the hollow would be perceived by a distant observer to run slightly more slowly than one just outside the hollow. The general expression computed [6] is $(\Delta t/t) = 2 \times 10^{-14}(0.2 R_\odot/R_\star)^{1/2}(M_\star/M_\odot)(1 \text{ pc}/D_\star)^{1/2}$. A clock placed in the gravitational hollow of Alpha Centauri A would therefore run about one part in $10^{-14}$ slower than a clock just outside the hollow. Since the NIST F1 clock keeps time to order one part in $10^{-15}$ [2], this time dilation is measurable. Other ways of detecting gravitational hollows are discussed by Nemiroff [6].

It is noted that General Relativity is inherently a non-linear theory where gravity is expected to affect gravity. One parameterization of this is the $\beta$ parameter in the PPN formalism [7]. We note that since our effect is dependant on distance from the gravitational lens (in the above falsifiable case, the Sun). The Sun’s effect is essentially negligible at the distance of the Earth. Therefore, $\beta$ does not have enough information in it to adequately describe this gravitational effect. It is therefore suggested here to parameterize this effect with the parameter $\psi$, with $\psi = 0$ showing a complete violation of the VEP, and $\psi = 1$ showing a complete conformity.

Radiations with wavelengths greater than the Schwarzschild radius of the lens undergo significant interference effects, effectively negating large gravitational lens magnifications. It is therefore possible that reality could produce $\psi$ values between zero and unity with the precise $\psi$ value a function of the mass of the lens itself.

The above example elucidates a test of quantum gravity that is practical but has never
been postulated before. The background spacetime on which quantum mechanics operates, and on which gravitational interactions occur, can be experimentally determined. This test can therefore open a new realm in the testing and understanding of gravity.