For problem 3.24 part (a) :

First, use the same trick we used for acceleration. That is,

$$\ddot{x} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) \quad \Rightarrow \ddot{\theta} = \frac{d}{d\theta}\left(\frac{1}{2}\dot{\theta}^2\right)$$

and integrate the equation from θ_0 to θ to get

$$\dot{\theta}^2 = 2(\cos\theta - \cos\theta_0)$$

or, taking the square root (only + matters) and rearranging

$$dt = \frac{d\theta}{\left(2\left(\cos\theta - \cos\theta_0\right)\right)^{1/2}}$$

Now integrate this from 0 to θ_0 to get 1/4-th of the period

$$\frac{T}{4} = \int_0^{\theta_0} \frac{d\theta}{\left(2\left(\cos\theta - \cos\theta_0\right)\right)^{1/2}}$$

and to get this into the form shown, you need to make the rather non-obvious substitutions

$$\alpha = \sin^2\left(\frac{\theta_0}{2}\right) \quad ; \quad \cos\theta = 1 - 2\alpha^2\sin^2\phi$$

from which you can show that

$$\cos\theta - \cos\theta_0 = 2\alpha^2\cos^2\phi$$
$$\sin\theta = 2\alpha\sin\phi\left(1-\alpha^2\sin^2\phi\right)^{1/2}$$

and $d\phi$ in terms of $d\theta$ is found from the second of these. Making this substitution, you get the integral shown. This integral is one type of "Elliptic Integral."