PH3111 Problems - Hamiltonian

1. Prob. 10.28

2. Prob. 10.29

3. Consider two (continuous) functions of generalized coordinates q_k and corresponding momenta, p_k, for example $g(q_k,p_k)$ and $h(q_k,p_k)$. The "Poisson Bracket" is defined by

$$[g,h] \equiv \sum_k \left(\frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_k} \right)$$

Show that:

(a) $$\frac{dg}{dt} = [g,H] + \frac{\partial g}{\partial t}$$

(b) $$[p_i, p_j] = [q_i, q_j] = 0$$

(c) $$[q_i, p_j] = \delta_{ij}$$

where H is the Hamiltonian and $\delta_{ij} = 1$ if $i = j$ and is zero otherwise. (Hint: if you are spending a lot of time on these, you are doing them wrong!)

Notes:

i. If $[g,h] = 0$, then $[g,h] = [h,g]$ and we say that "g and h commute" (for this operation).

ii. If $[g,h] = 1$, then g and h are said to be "canonically conjugate."

iii. (a) implies that if the quantity represented by g is not an explicit function of time, then it will be a constant of the motion as long as $[g,H] = 0$, that is, if g and H commute. (Notice that H may be a function of time!)

iv. (a) can be used with $g = p_k$ and with $g = q_k$, along with the fact that our coordinates are not explicit functions of time, to show that

$$\dot{q}_j = [q_j, H], \qquad \dot{p}_j = [p_j, H]$$