PH2100 Formula Sheet Knight

Concepts of Motion

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}}$$

$$\Delta\vec{r} = \vec{r}_\text{f} - \vec{r}_\text{i}$$

$$\vec{v}_\text{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{a}_\text{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Kinematics: The Mathematics of Motion

For Motion in a Straight Line:

$$v_s = \frac{ds}{dt} = \text{slope of position-time graph}$$

$$a_s = \frac{dv_s}{dt} = \text{slope of velocity-time graph}$$

$$s_\text{f} = s_\text{i} + \int_{t_\text{i}}^{t_\text{f}} v_s\,dt = s_\text{i} + \begin{cases}\text{area under velocity curve} \\ \text{from } t_\text{i} \text{ to } t_\text{f}\end{cases}$$

$$v_\text{fs} = v_\text{is} + \int_{t_\text{i}}^{t_\text{f}} a_s\,dt = v_\text{is} + \begin{cases}\text{area under acceleration curve} \\ \text{from } t_\text{i} \text{ to } t_\text{f}\end{cases}$$

Uniform Motion:

$$s_\text{f} = s_\text{i} + v_s\Delta t$$

Uniformly Accelerated Motion:

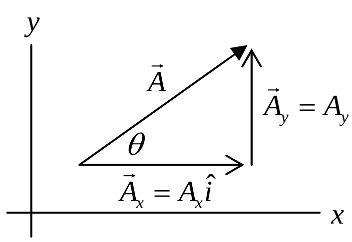
$$v_\text{fs} = v_\text{is} + a_s\Delta t$$

$$s_\text{f} = s_\text{i} + v_\text{is}\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$

$$v_\text{fs}{}^2 = v_\text{is}{}^2 + 2a_s\Delta s$$

Motion on an Inclined Plane: $a_s = \pm g\sin\theta$

Vectors and Coordinate Systems



$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$$

In the figure above:

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2} \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Force and Motion

$$\vec{a} = \frac{1}{m}\vec{F}_\text{net}$$

where $\vec{F}_\text{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$

Dynamics I: Motion Along a Line

$$\vec{F}_\text{net} = \sum_i \vec{F}_i = m\vec{a}$$

A Model for Friction:

Static: $\vec{f}_\text{s} \le (\mu_\text{s} n, \text{ direction as necessary to prevent motion})$

Kinetic: $\vec{f}_\text{k} = (\mu_\text{k} n, \text{ direction opposite the motion})$

Rolling: $\vec{f}_\text{r} = (\mu_\text{r} n, \text{ direction opposite the motion})$

Drag: $\vec{D} \approx \left(\tfrac{1}{4}Av^2, \text{ direction opposite the motion}\right)$

Dynamics II: Motion in a Plane

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

$$(F_\text{net})_x = \sum F_x = ma_x$$

$$(F_\text{net})_y = \sum F_y = ma_y$$

Constant Acceleration:

$$x_\text{f} = x_\text{i} + v_\text{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_\text{f} = y_\text{i} + v_\text{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_\text{fx} = v_\text{ix} + a_x\Delta t \qquad v_\text{fy} = v_\text{iy} + a_y\Delta t$$

Projectile Motion:

$$x_\text{f} = x_\text{i} + v_\text{ix}\Delta t \qquad y_\text{f} = y_\text{i} + v_\text{iy}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$v_\text{fx} = v_\text{ix} = \text{constant} \qquad v_\text{fy} = v_\text{iy} - g\Delta t$$

$$v_\text{fy}^2 = v_\text{iy}^2 - 2g(y_\text{f} - y_\text{i})$$

Relative Motion:

$$\vec{r} = \vec{r}\,' + \vec{V}t$$

$$x = x' + V_x t$$

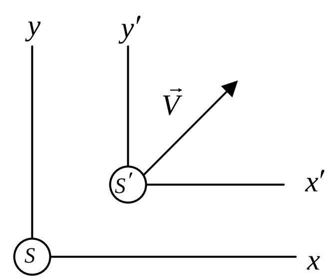
$$y = y' + V_y t$$

$$\vec{v} = \vec{v}\,' + \vec{V}$$

$$v_x = v'_x + V_x$$

$$v_y = v'_y + V_y$$

$$\vec{a} = \vec{a}\,'$$



Need additional help? Then visit the Physics Learning Center located in 228 Fisher. Walk-in hours are Sunday 7:00 – 9:00 p.m., Monday 3:00 – 5:00 and 6:00 – 9:00 p.m., and Tuesday through Thursday 3:00 - 9:00 p.m.

Dynamics III: Motion in a Circle

$$\theta = \frac{s}{r}$$

$$\text{average angular velocity} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

$$v_t = \omega r$$

Uniform Circular Motion:

$$v = \frac{2\pi r}{T} \quad \omega = \frac{2\pi \text{ rad}}{T}$$

$$\theta_{\text{f}} = \theta_{\text{i}} + \omega\ \Delta t$$

$$a_r = \frac{v^2}{r} = \omega^2 r$$

Nonuniform Circular Motion (constant a_t):

$$a_t = \frac{dv}{dt}$$

$$\theta_{\text{f}} = \theta_{\text{i}} + \omega_{\text{i}}\Delta t + \frac{a_t}{2r}(\Delta t)^2$$

$$\omega_{\text{f}} = \omega_{\text{i}} + \frac{a_t}{r}\Delta t$$

$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

$$(F_{\text{net}})_t = \sum F_t = \begin{cases} 0 & \text{uniform motion} \\ ma_t & \text{nonuniform motion} \end{cases}$$

$$(F_{\text{net}})_z = \sum F_z = 0$$

Newton's Third Law

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

Impulse and Momentum

$$\vec{p} = m\vec{v}$$

$$J_x = \begin{cases} \int_{t_{\text{i}}}^{t_{\text{f}}} F_x(t)dt = \text{area under force curve} \\ F_{\text{avg}}\Delta t \end{cases}$$

$$\Delta p_x = J_x \text{ (impulse-momentum theorem)}$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$

$$\vec{P}_{\text{f}} = \vec{P}_{\text{i}} \text{ (conservation of momentum)}$$

Energy

$$K = \tfrac{1}{2}mv^2$$

$$U_{\text{g}} = mgy$$

$$E_{\text{mech}} = K + U$$

$$K_{\text{f}} + U_{\text{f}} = K_{\text{i}} + U_{\text{i}} \text{ (conservation of mechanical energy)}$$

$$(F_{\text{sp}})_s = -k\Delta s \qquad U_s = \tfrac{1}{2}k(\Delta s)^2$$

Perfectly Elastic 1-D Collisions (m_2 initially at rest):

$$(v_{\text{f}x})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{\text{i}x})_1 \qquad (v_{\text{f}x})_2 = \frac{2m_1}{m_1 + m_2}(v_{\text{i}x})_1$$

Work

$$W = \begin{cases} \int_{s_{\text{i}}}^{s_{\text{f}}} F_s ds = \text{area under the force-position curve} \\ \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta \quad \text{if } \vec{F} \text{ is a constant force} \end{cases}$$

$$\Delta K = W_{\text{net}} = W_{\text{c}} + W_{\text{diss}} + W_{\text{ext}} \text{ (work-kinetic energy theorem)}$$

$$\Delta U = U_{\text{f}} - U_{\text{i}} = -W_{\text{c}}(\text{i} \to \text{f})$$

$$F_s = -\frac{dU}{ds}$$

$$E_{\text{th}} = K_{\text{micro}} + U_{\text{micro}}$$

$$\Delta E_{\text{th}} = -W_{\text{diss}}$$

$$E_{\text{sys}} = K + U + E_{\text{th}}$$

$$K_{\text{f}} + U_{\text{f}} + \Delta E_{\text{th}} = K_{\text{i}} + U_{\text{i}} + W_{\text{ext}}$$

$$P = \frac{dE_{\text{sys}}}{dt} \qquad P = \frac{dW}{dt} = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

Newton's Theory of Gravity

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

$$g_{\text{surface}} = \frac{GM}{R^2}$$

$$\text{Circular Orbit: } v = \sqrt{\frac{GM}{r}} \quad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

Physical Constants

$$g = 9.80 \text{ m/s}^2$$

$$G = 6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

$$M_{\text{earth}} = 5.98\times10^{24} \text{ kg}$$

$$R_{\text{earth}} = 6.37\times10^{6} \text{ m}$$

Need additional help? Then visit the Physics Learning Center located in 228 Fisher. Walk-in hours are Sunday 7:00 – 9:00 p.m., Monday 3:00 – 5:00 and 6:00 – 9:00 p.m., and Tuesday through Thursday 3:00 - 9:00 p.m.