

## Concepts of Motion

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}}$$

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}$$

## Kinematics: The Mathematics of Motion

For Motion in a Straight Line:

$$v_s = \frac{ds}{dt} = \text{slope of position-time graph}$$

$$a_s = \frac{dv_s}{dt} = \text{slope of velocity-time graph}$$

$$s_f = s_i + \int_{t_i}^{t_f} v_s dt = s_i + \begin{cases} \text{area under velocity curve} \\ \text{from } t_i \text{ to } t_f \end{cases}$$

$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s dt = v_{is} + \begin{cases} \text{area under acceleration curve} \\ \text{from } t_i \text{ to } t_f \end{cases}$$

Uniform Motion:

$$s_f = s_i + v_s \Delta t$$

Uniformly Accelerated Motion:

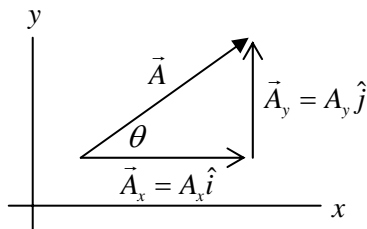
$$v_{fs} = v_{is} + a_s \Delta t$$

$$s_f = s_i + v_{is} \Delta t + \frac{1}{2} a_s (\Delta t)^2$$

$$v_{fs}^2 = v_{is}^2 + 2a_s \Delta s$$

Motion on an Inclined Plane :  $a_s = \pm g \sin \theta$

## Vectors and Coordinate Systems



$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j}$$

In the figure above:

$$A_x = A \cos \theta \quad A_y = A \sin \theta$$

$$A = \sqrt{A_x^2 + A_y^2} \quad \theta = \tan^{-1} \left( \frac{A_y}{A_x} \right)$$

## Force and Motion

$$\vec{a} = \frac{1}{m} \vec{F}_{\text{net}}$$

$$\text{where } \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

## Dynamics I: Motion Along a Line

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

A Model for Friction :

Static :  $\vec{f}_s \leq (\mu_s n, \text{ direction as necessary to prevent motion})$

Kinetic :  $\vec{f}_k = (\mu_k n, \text{ direction opposite the motion})$

Rolling :  $\vec{f}_r = (\mu_r n, \text{ direction opposite the motion})$

Drag :  $\vec{D} \approx \left( \frac{1}{4} A v^2, \text{ direction opposite the motion} \right)$

## Dynamics II: Motion in a Plane

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

$$(F_{\text{net}})_x = \sum F_x = ma_x$$

$$(F_{\text{net}})_y = \sum F_y = ma_y$$

Constant Acceleration :

$$x_f = x_i + v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \quad y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x \Delta t$$

$$v_{fy} = v_{iy} + a_y \Delta t$$

Projectile Motion :

$$x_f = x_i + v_{ix} \Delta t$$

$$v_{fx} = v_{ix} = \text{constant}$$

$$y_f = y_i + v_{iy} \Delta t - \frac{1}{2} g (\Delta t)^2$$

$$v_{fy} = v_{iy} - g \Delta t$$

$$v_{fy}^2 = v_{iy}^2 - 2g(y_f - y_i)$$

Relative Motion:

$$\vec{r} = \vec{r}' + \vec{V}t$$

$$x = x' + V_x t$$

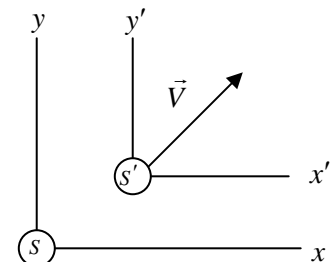
$$y = y' + V_y t$$

$$\vec{v} = \vec{v}' + \vec{V}$$

$$v_x = v'_x + V_x$$

$$v_y = v'_y + V_y$$

$$\vec{a} = \vec{a}'$$



Need additional help? Then visit the Physics Learning Center located in 228 Fisher. Walk-in hours are Sunday 7:00 – 9:00 p.m., Monday 3:00 – 5:00 and 6:00 – 9:00 p.m., and Tuesday through Thursday 3:00 – 9:00 p.m.

## Dynamics III: Motion in a Circle

$$\theta = \frac{s}{r}$$

$$\text{average angular velocity} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

$$v_t = \omega r$$

Uniform Circular Motion :

$$v = \frac{2\pi r}{T} \quad \omega = \frac{2\pi \text{ rad}}{T}$$

$$\theta_f = \theta_i + \omega \Delta t$$

$$a_r = \frac{v^2}{r} = \omega^2 r$$

Nonuniform Circular Motion (constant  $a_t$ ):

$$a_t = \frac{dv}{dt}$$

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{a_t}{2r} (\Delta t)^2$$

$$\omega_f = \omega_i + \frac{a_t}{r} \Delta t$$

$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

$$(F_{\text{net}})_t = \sum F_t = \begin{cases} 0 & \text{uniform motion} \\ ma_t & \text{nonuniform motion} \end{cases}$$

$$(F_{\text{net}})_z = \sum F_z = 0$$

## Newton's Third Law

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

## Impulse and Momentum

$$\vec{p} = m\vec{v}$$

$$J_x = \begin{cases} \int_{t_i}^{t_f} F_x(t) dt = \text{area under force curve} \\ F_{\text{avg}} \Delta t \end{cases}$$

$$\Delta p_x = J_x \text{ (impulse-momentum theorem)}$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$$

$$\vec{P}_f = \vec{P}_i \text{ (conservation of momentum)}$$

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## Energy

$$K = \frac{1}{2}mv^2$$

$$U_g = mgy$$

$$E_{\text{mech}} = K + U$$

$$K_f + U_f = K_i + U_i \text{ (conservation of mechanical energy)}$$

$$(F_{\text{sp}})_s = -k\Delta s \quad U_s = \frac{1}{2}k(\Delta s)^2$$

Perfectly Elastic 1-D Collisions ( $m_2$  initially at rest):

$$(v_{\text{fx}})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{\text{ix}})_1 \quad (v_{\text{fx}})_2 = \frac{2m_1}{m_1 + m_2} (v_{\text{ix}})_1$$

## Work

$$W = \begin{cases} \int_{s_i}^{s_f} F_s ds = \text{area under the force-position curve} \\ \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos \theta \quad \text{if } \vec{F} \text{ is a constant force} \end{cases}$$

$$\Delta K = W_{\text{net}} = W_c + W_{\text{diss}} + W_{\text{ext}} \text{ (work-kinetic energy theorem)}$$

$$\Delta U = U_f - U_i = -W_c \text{ (i} \rightarrow \text{f)}$$

$$F_s = -\frac{dU}{ds}$$

$$E_{\text{th}} = K_{\text{micro}} + U_{\text{micro}}$$

$$\Delta E_{\text{th}} = -W_{\text{diss}}$$

$$E_{\text{sys}} = K + U + E_{\text{th}}$$

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i + W_{\text{ext}}$$

$$P = \frac{dE_{\text{sys}}}{dt} \quad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

## Newton's Theory of Gravity

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

$$g_{\text{surface}} = \frac{GM}{R^2}$$

$$\text{Circular Orbit: } v = \sqrt{\frac{GM}{r}} \quad T^2 = \left( \frac{4\pi^2}{GM} \right) r^3$$

## Physical Constants

$$g = 9.80 \text{ m/s}^2$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$

$$M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$R_{\text{earth}} = 6.37 \times 10^6 \text{ m}$$