

Concepts of Motion

average speed = $\frac{\text{distance traveled}}{\text{time interval spent traveling}}$

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}$$

Kinematics in One Dimension

For Motion in a Straight Line:

$$v_s = \frac{ds}{dt} = \text{slope of position-time graph}$$

$$a_s = \frac{dv_s}{dt} = \text{slope of velocity-time graph}$$

$$s_f = s_i + \int_{t_i}^{t_f} v_s dt = s_i + \left\{ \begin{array}{l} \text{area under velocity curve} \\ \text{from } t_i \text{ to } t_f \end{array} \right.$$

$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s dt = v_{is} + \left\{ \begin{array}{l} \text{area under acceleration curve} \\ \text{from } t_i \text{ to } t_f \end{array} \right.$$

Uniform Motion:

$$s_f = s_i + v_s \Delta t$$

Uniformly Accelerated Motion:

$$v_{fs} = v_{is} + a_s \Delta t$$

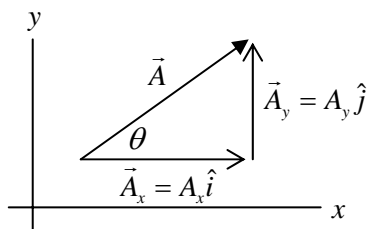
$$s_f = s_i + v_{is} \Delta t + \frac{1}{2} a_s (\Delta t)^2$$

$$v_{fs}^2 = v_{is}^2 + 2a_s \Delta s$$

Free fall: $a_y = -g = -9.80 \text{ m/s}^2$

Motion on an Inclined Plane: $a_s = \pm g \sin \theta$

Vectors and Coordinate Systems



$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j}$$

In the figure above:

$$A_x = A \cos \theta \quad A_y = A \sin \theta$$

$$A = \sqrt{A_x^2 + A_y^2} \quad \theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

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Kinematics in Two Dimensions

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

Constant Acceleration:

$$x_f = x_i + v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \quad y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x \Delta t$$

$$v_{fy} = v_{iy} + a_y \Delta t$$

Projectile Motion:

$$x_f = x_i + v_{ix} \Delta t$$

$$y_f = y_i + v_{iy} \Delta t - \frac{1}{2} g (\Delta t)^2$$

$$v_{fx} = v_{ix} = \text{constant}$$

$$v_{fy} = v_{iy} - g \Delta t$$

$$v_{fy}^2 = v_{iy}^2 - 2g(y_f - y_i)$$

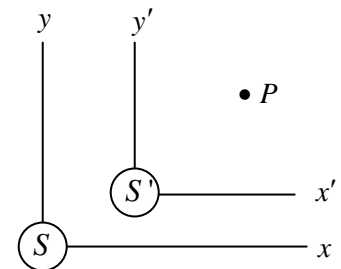
Relative Motion:

$$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{v}_{S'S} t$$

$$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$$



$$\vec{a}_{PS} = \vec{a}_{PS'}$$



Circular Motion:

$$\theta = \frac{s}{r}$$

$$\text{average angular velocity} = \frac{\Delta \theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

$$v_t = \omega r$$

$$\alpha = \frac{d\omega}{dt}$$

Uniform Circular Motion (constant ω):

$$v_t = \frac{2\pi r}{T} \quad \omega = \frac{2\pi \text{ rad}}{T}$$

$$\theta_f = \theta_i + \omega \Delta t$$

$$a_r = \frac{v_t^2}{r} = \omega^2 r$$

Nonuniform Circular Motion (constant a_t):

$$a_t = \frac{dv_t}{dt} = \alpha r$$

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta$$

Force and Motion

$$\vec{a} = \frac{1}{m} \vec{F}_{\text{net}}$$

$$\text{where } \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

Dynamics I: Motion Along a Line

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

$$\vec{F}_G = (mg, \text{downward})$$

$$w = mg \left(1 + \frac{a_y}{g} \right)$$

A Model for Friction :

$$\text{Static: } \vec{f}_s = \begin{cases} 0 \text{ to } \mu_s n \\ \text{direction as necessary to prevent motion} \end{cases}$$

$$\text{Kinetic: } \vec{f}_k = (\mu_k n, \text{direction opposite the motion})$$

$$\text{Rolling: } \vec{f}_r = (\mu_r n, \text{direction opposite the motion})$$

$$\text{Drag: } \vec{D} \approx \left(\frac{1}{4} A v^2, \text{direction opposite the motion} \right)$$

Newton's Third Law

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

Dynamics II: Motion in a Plane

$$(F_{\text{net}})_x = \sum F_x = ma_x$$

$$(F_{\text{net}})_y = \sum F_y = ma_y$$

$$(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$$

$$(F_{\text{net}})_t = \sum F_t = \begin{cases} 0 & \text{uniform motion} \\ ma_t & \text{nonuniform motion} \end{cases}$$

$$(F_{\text{net}})_z = \sum F_z = 0$$

Impulse and Momentum

$$\vec{p} = m\vec{v}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$J_x = \begin{cases} \int_{t_i}^{t_f} F_x(t) dt = \text{area under force curve} \\ F_{\text{avg}} \Delta t \end{cases}$$

$$\Delta p_x = J_x \text{ (impulse-momentum theorem)}$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$$

$$\vec{P}_f = \vec{P}_i \text{ (conservation of momentum)}$$

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Energy

$$K = \frac{1}{2} mv^2$$

$$U_g = mgy$$

$$E_{\text{mech}} = K + U$$

$$K_f + U_f = K_i + U_i \text{ (conservation of mechanical energy)}$$

$$(F_{\text{sp}})_s = -k\Delta s \quad U_s = \frac{1}{2} k (\Delta s)^2$$

Perfectly Elastic 1-D Collisions (m_2 initially at rest):

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1 \quad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

Work

$$W = \begin{cases} \int_{s_i}^{s_f} F_s ds = \text{area under the force-position curve} \\ \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos \theta \quad \text{if } \vec{F} \text{ is a constant force} \end{cases}$$

$$\Delta K = W_{\text{net}} = W_c + W_{\text{diss}} + W_{\text{ext}} \text{ (work-kinetic energy theorem)}$$

$$\Delta U = U_f - U_i = -W_c \text{ (i} \rightarrow \text{f)}$$

$$F_s = -\frac{dU}{ds}$$

$$E_{\text{th}} = K_{\text{micro}} + U_{\text{micro}}$$

$$\Delta E_{\text{th}} = -W_{\text{diss}} = f_k \Delta s$$

$$E_{\text{sys}} = K + U + E_{\text{th}}$$

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i + W_{\text{ext}}$$

$$P = \frac{dE_{\text{sys}}}{dt} \quad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

Newton's Theory of Gravity

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2}$$

$$g_{\text{surface}} = \frac{GM}{R^2}$$

$$\text{Circular Orbit: } v = \sqrt{\frac{GM}{r}} \quad T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

Physical Constants

$$g = 9.80 \text{ m/s}^2$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$

$$M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$$

$$R_{\text{earth}} = 6.37 \times 10^6 \text{ m}$$