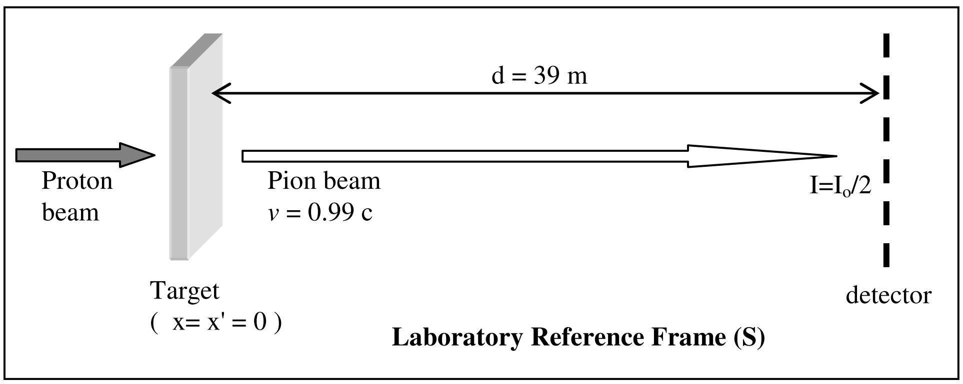
Homework 11. Relativity Problems PH3110 Fall 2006 Due 12/6/06

1. F&C 5.13

2. Complete the time dilation derivation we started in class based on the light reflecting off of the mirror experiment. Show that $\Delta t = \dfrac{\Delta t'}{\sqrt{1 - v^2/c^2}}$.

3. Pions have a half-life of 1.77×10^{-8} s. That is, half of the pions (at rest) present at any time will have decayed 1.77×10^{-8} s later. Pions can be generated by accelerating a beam of protons at high speeds into a suitable target material. Consider an experiment in which a collimated beam of high-energy pions is moving away from the source target with speed v = 0.99 c. It is found that the beam drops in intensity by one half at a distance of 39 m from the target.



(a) If the halflife in the laboratory frame is the same as the halflife in the pion's rest frame, what fraction of the initial intensity would you expect at 39 m from the target? Is this measurably different from the $I_o/2$ actually measured at the target?
(b) Again, if the halflife in the laboratory frame is the same as the halflife in the pion's rest frame, at what distance from the target would you expect the intensity of the pion beam to be $I_o/2$? Is this consistent with the observed 39 m?
(c) Show that time dilation can reconcile the measurements. Do this from the laboratory reference frame, S.
(d) Show that length contraction can also reconcile the measurements by looking at the situation from the reference frame of the pions, S'.

4. A 100-MeV electron for which $\beta \equiv v/c = 0.999975$ moves along the axis of an evacuated tube that has a proper length 3.00 m. An observer S moving with the electron would see the tube moving past at a speed v, but in the opposite direction. What length would observer S measure for this tube?

5. The lifetime of μ-mesons stopped in a lead block in the laboratory is measured to be 2.3×10^{-6} s. The mean lifetime of high-speed μ-mesons in a burst of cosmic rays observed from the earth's surface is measured to be 1.6×10^{-5} s. find the speed of the μ-mesons.

6. Complete the derivation in class of the phase difference for time between two points in space in frame S' according to observer S: that is, show that $\Delta t' = L'v/c^2$.

7. For our usual two reference frames, S, S', where S' moves with velocity in the +x (+x') direction, the Lorentz transformation equations that relate observations in S' to what an observer would measure in S are:

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + (v/c^2)x'}{\sqrt{1 - v^2/c^2}}$$

Start from this S' to S transformation and find the transformation from S to S', that is, show that

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

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$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

8. Two electrons each leave a radioactive sample in opposite directions at speed 0.67 c with respect to the sample at rest in the laboratory. According to classical physics, the speed of one electron relative to the other should be 1.34 c. Note that this is greater than c! What is the relativistic result for the speed of one electron relative to the other?

9. Extra Credit: In special relativity we confine our reference frames to be inertial, but we can still describe objects accelerating in those reference frames. Starting from the relativistic velocity transformation formula in the x-direction, derive the relativistic acceleration transformation formula for the x-direction by differentiation.

$$a'_x = \frac{a_x \left(1 - v^2/c^2\right)^{3/2}}{\left(1 - u_x v/c^2\right)^3}$$

[Hint: $a_x = du_x/dt$, $a'_x = du'_x/dt'$, and by the chain rule, $d/dt' = (dt/dt')d/dt$.]

Note that the acceleration depends on the inertial frame for observation of the accelerating object, in contrast to the Gallilean Transformation. What affect might you guess this has on Newton's 2nd Law as commonly written **F**=m**a**?

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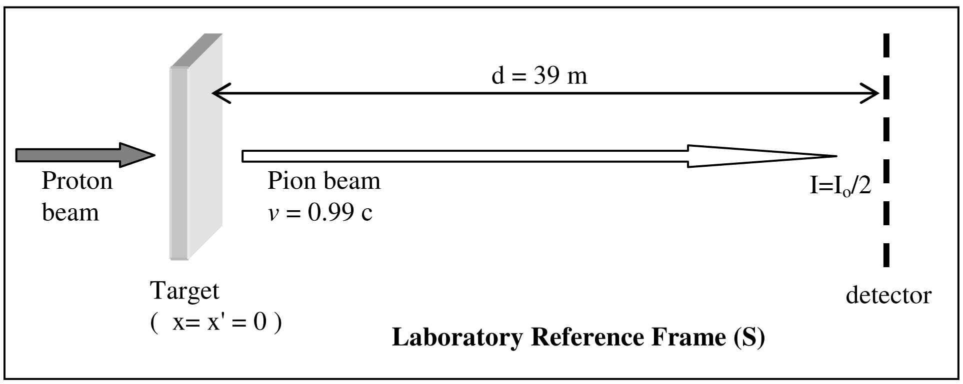

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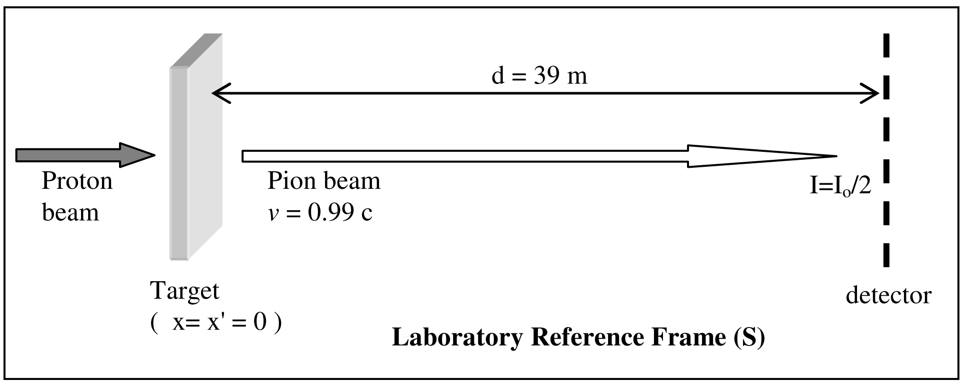

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