Homework 5. Ch. 3 Problems PH3110 Fall 2006 Due 10/11/06

1. In 1901 MTU president and physicist extraordinaire Fred W. McNair and colleagues tested a pendulum 4,250 feet long down the #5 shaft of the Tamarack mine in Calumet. They hung a mass of 50 pounds at the bottom. Ignoring the mass of the piano wire holding the 50-lb mass (which in fact is significant and makes the overall center-of-mass of the system quite high above the hanging mass), what is the expected period of oscillation of the pendulum swinging with small amplitude?
2. As shown in class and in the book, when a mass hangs on a vertical spring at rest, it has a new equilibrium position. Write a simple expression for the total potential energy of a mass hanging on a vertical spring taking into account both V_{spring} and $V_{gravity}$. Show that apart from a constant term, the potential energy can be written as a quadratic function of the displacement from the new equilibrium position.
3. Show that in an overdamped harmonic mass-spring system, the mass can pass through the origin at most one time. To do this, suppose at some time x=0. We are free to call this time t=0. At this time let it be moving with some speed v_o in the positive direction (no loss of generality). Write down the initial conditions for x(0) and v(0) that fix the constants C_1 and C_2. Solve for these and write the appropriate expression for x(t). Plot this using dimensionless variables (which you need to define) using a parameterization $\gamma=4\omega_o$. (Note: the amplitude A is a perfectly good parameter to use in defining define x'.) What happens after x reaches a maximum?
4. Consider the critically damped mass-spring system. Now take the initial conditions that x(0)=0 and $v(0)=v_0$. Solve for the constants A and B and find an expression for x(t). Plot it using dimensionless variables on the same graph as for problem 3 if you can.
5. F&C 3.1
6. F&C 3.7