PH2100 Formula Sheet

Motion in One Dimension

$$average\ speed \equiv \frac{total\ distance}{total\ time}$$

$$\Delta x \equiv x_f - x_i$$

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t}$$

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t}$$

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$v_{xf} = v_{xi} + a_x t$$

$$x_f - x_i = v_{xi} t + \tfrac{1}{2} a_x t^2$$

$$x_f - x_i = \bar{v}_x t = \tfrac{1}{2}(v_{xi} + v_{xf}) t$$

$$v_{xf}{}^2 = v_{xi}{}^2 + 2a_x (x_f - x_i)$$

Vectors

$$A_x = A\cos q$$

$$A_y = A\sin q$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$q = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Motion in Two Dimensions

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\Delta\vec{r} \equiv \vec{r}_f - \vec{r}_i$$

$$\vec{v}_{avg} \equiv \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{a}_{avg} \equiv \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$

$$y = (\tan q_i)x - \left(\frac{g}{2v_i^2 \cos^2 q_i}\right)x^2$$

$$h = \frac{v_i^2 \sin^2 q_i}{2g} \qquad R = \frac{v_i^2 \sin 2q_i}{g}$$

$$a_r = \frac{v^2}{r}, \quad a_t = \frac{dv}{dt}$$

$$\vec{a} = \frac{d|\vec{v}|}{dt}\hat{q} - \frac{v^2}{r}\hat{r}$$

$$\vec{v}\,' = \vec{v} - \vec{v}_0$$

$$\vec{a}\,' = \vec{a}$$

The Laws of Motion

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_g = m\vec{g}$$

$$f_k = m_k n$$

$$f_s \leq m_s n$$

$$F_{spring} = -kx$$

Circular Motion

$$\sum F_r = ma_r = \frac{mv^2}{r}$$

Work and Kinetic Energy

$$W \equiv Fd\cos q$$

$$\vec{A}\cdot\vec{B} \equiv AB\cos q = A_x B_x + A_y B_y + A_z B_z$$

$$W = \int_{x_i}^{x_f} F_x dx$$

$$K \equiv \tfrac{1}{2}mv^2$$

$$\sum W = \Delta K$$

$$K_i + \sum W_{other} - f_k d = K_f$$

$$\bar{\mathbf{P}} \equiv \frac{W}{\Delta t}$$

$$\mathbf{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$

Potential Energy and Energy Conservation

$$\Delta U = U_f - U_i \equiv -\int_{x_i}^{x_f} F_x dx$$

$$U_g = mgy$$

$$U_s = \tfrac{1}{2}kx^2$$

$$E \equiv K + U$$

$$K_i + \sum U_i = K_f + \sum U_f$$

$$W_{app} = \Delta K + \Delta U$$

$$F_x = -\frac{dU}{dx}$$

Linear Momentum and Collisions

$$\vec{p} \equiv m\vec{v}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$\vec{I} \equiv \int_{t_i}^{t_f} \vec{F}dt = \Delta\vec{p}$$

$$\vec{F}_{avg} = \frac{\vec{I}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

$$\vec{r}_{CM} \equiv \frac{\sum m_i \vec{r}_i}{M}$$

$$\vec{v}_{CM} \equiv \frac{d\vec{r}_{CM}}{dt} = \frac{\sum m_i \vec{v}_i}{M}$$

$$\vec{a}_{CM} \equiv \frac{d\vec{v}_{cm}}{dt} = \frac{\sum m_i a_i}{M}$$

$$\sum \vec{F}_{ext} = M\vec{a}_{CM} = \frac{d\vec{p}_{tot}}{dt}$$

Rotation About a Fixed Axis

$$s = rq$$

$$\bar{w} \equiv \frac{\Delta q}{\Delta t}$$

$$w \equiv \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

$v = w r$

$\bar{a} \equiv \frac{\Delta w}{\Delta t}$

$a \equiv \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw}{dt}$

$a_t = \frac{dv}{dt} = r a$

$a_r = \frac{v^2}{r} = w^2 r$

$w_f = w_i + a t$

$q_f = q_i + w_i t + \frac{1}{2} a t^2$

$q_f = q_i + \frac{1}{2}(w_i + w_f) t$

$w_f^{\,2} = w_i^2 + 2a(q_f - q_i)$

$I = \sum m_i r_i^2$

$I = I_{CM} + MD^2$

$K_R = \frac{1}{2} I w^2$

$t \equiv rF \sin f = Fd$

$\sum t_{ext} = Ia$

$\mathbf{P} = t w$

$\sum W = \frac{1}{2} I w_f^{\,2} - \frac{1}{2} I w_i^{\,2}$

Rolling Motion and Angular Momentum

$K = \frac{1}{2} I_{CM} w^2 + \frac{1}{2} M v_{CM}^{\;\,2}$

$v_{CM} = Rw$

$\vec{a} \equiv \frac{d\vec{w}}{dt}$

$\vec{t} \equiv \vec{r} \times \vec{F}$

$\vec{L} \equiv \vec{r} \times \vec{p}$

$|\vec{A} \times \vec{B}| = AB \sin q$

$L_z = Iw$

$\sum \vec{t}_{ext} = \frac{d\vec{L}}{dt}$

$\sum \vec{t}_{ext} = 0 \Rightarrow \vec{L} = \text{constant}$

Static Equilibrium

$\sum \vec{F} = 0$

$\sum \vec{t} = 0$

The Law of Gravity

$F_g = \frac{G m_1 m_2}{r^2}$

$g' = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E + h)^2}$

$T^2 = \frac{4p^2 r^3}{GM}$

$\vec{g} \equiv \frac{\vec{F}_g}{m}$

$U = -\frac{G m_1 m_2}{r}$

$E = \frac{1}{2} m v^2 - \frac{GMm}{r}$

$E = -\frac{GMm}{2r}$

$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$

Simple Harmonic Motion

$x = A\cos(wt + f)$

$v = \pm w\sqrt{A^2 - x^2}$

$f = \frac{w}{2p}$

$T = \frac{1}{f} = \frac{2p}{w}$

$w = \sqrt{\frac{k}{m}}$

$w = \sqrt{\frac{k}{I}}$

$w = \sqrt{\frac{g}{L}}$

$w = \sqrt{\frac{mgd}{I}}$

$E = \frac{1}{2} k A^2$

Wave Motion

$y = f(x \pm vt)$

$v = \sqrt{T/m}$

$y = A\sin(kx - wt)$

$v = l f$

$k \equiv 2p/l \qquad w \equiv 2p f$

$\mathbf{P} = \frac{1}{2} m w^2 A^2 v$

Sound Waves

$I = \frac{\mathbf{P}_{av}}{A} = \frac{\mathbf{P}_{av}}{4p r^2}$

$b = 10\log(I/I_o)$

$f' = \left(\frac{v \pm v_o}{v \mp v_s}\right) f \quad \left(\frac{upper : approach}{lower : recession}\right)$

$\sin q = \frac{v}{v_s}$

Superposition and Standing Waves

$\Delta r = \frac{f}{2p} l$

$f_n = n \frac{v}{2L} \qquad n = 1,2,3,...$

$f_n = n \frac{v}{4L} \qquad n = 1,3,5,...$

$f_b = |f_1 - f_2|$

Physical Constants

$g = 9.80\ m/s^2$

$G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$

$M_{Earth} = 5.98 \times 10^{24}\ kg$

$R_{Earth} = 6.37 \times 10^{6}\ m$

$N_A = 6.02 \times 10^{23}$

$I_o = 1.00 \times 10^{-12}\ W/m^2$