Direct Observations of Coherent Backscatter of Radar Waves in Precipitation

A. R. JAMESON

RJH Scientific, Inc., El Cajon, California

A. B. KOSTINSKI

Michigan Technological University, Houghton, Michigan

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ABSTRACT

In previous work, it was argued that a source of radar coherent scatter occurs in the direction perpendicular to the direction of wave propagation because of the presence of grids of enhanced particle concentrations with spatial periodicities in resonance with the radar wavelength. While convincing, the evidence thus far has been indirect. In this work the authors now present direct observations of radar coherent backscattered signals in precipitation in the direction of wave propagation.

The theory is developed for the cross-correlation function of the complex amplitudes in the direction of propagation calculated for nearest neighbor range bins. Data are analyzed in snow and in rain. The results agree with the earlier conclusions in the previous work, namely that coherent scatter occurs in both rain and snow, that it is larger in snow than it is in rain, and that it can be significant at times.

1. Introduction

In previous studies (Jameson and Kostinski 2010a, hereafter JK10a) the presence of radar coherent signals backscattered by precipitation was inferred from the temporal spectral characteristics of the backscattered signals. In that work (see Fig. 7 in JK10a), it was reported that on average 72% and 34% of the power from rain and snow, respectively, arose from coherent scatter. However, the conclusions in JK10a were inferential, leaving some doubtful skeptics. To address these doubts, we provide in this work direct evidence of such scatter using the cross-correlation function of the complex amplitudes between neighboring range bins averaged over time.

The classical autocorrelation functions (AC) of complex amplitudes in time are always taken at each range bin independently of the other range bins. Indeed, from the classical perspective the complex amplitudes at each range bin should, in time, be statistically independent of those in neighboring bins when the scatter is incoherent. Hence, the fluctuating components at each bin arising from differential particle velocities imply that time averaging of a cross correlation between range bins should always average to zero. Indeed, it is shown below theoretically that this is true when there is no spatial correlation *on any scale* among the scatterers over a large domain (a near impossibility in the atmosphere), but it is not true when there are spatial correlations of the structures of the precipitation on scales of the radar wavelength.

If the scales of these structures are in resonance with the wavelength in a direction orthogonal to the direction of propagation of the transmitted wave, JK10a have already argued that coherent backscatter likely occurs. But what about in the direction of propagation?

Some investigators have argued against the presence of coherent scatter by combining observations in neighboring range bins. In statistically homogeneous conditions and when only incoherent scatter is present, it is argued that the total power from the combination of both bins should double when the volume is doubled because the total number of particles N doubles. In contrast, because coherent scatter goes as N^2, it is mistakenly argued that the total power of the combined bins should increase at a rate greater than linear. Since that is not seen, it has been concluded by some that coherent scatter does not

Corresponding author address: A. R. Jameson, 5625 N. 32nd Street, Arlington, VA 22207–1560.
E-mail: arjatrjhsci@verizon.net

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exist. This, however, is a fallacious argument for two reasons. First, if the backscattered power in each bin is already dominated by coherent scatter, combining two such bins will simply yield twice the total power. Second, even if one were to combine the complex amplitudes in each bin before computing the total power, the presence of any coherent scatter could not be detected since the addition of the two complex amplitudes would look like any other complex amplitude associated with twice the power. Hence, such approaches are ineffective.

However, instead, we show below that there is a more direct method for observing coherent scatter as a function of radar range. That is, we consider the spatial cross-correlation functions between neighboring range bins. The primary purpose of this paper, then, is to develop the theoretical expressions for the complex amplitude cross-correlation functions when both incoherent and coherent backscatter are present. We then show that statistically meaningful real values of time-averaged cross correlations only occur when there are spatial structures in resonance with the radar wavelength (i.e., where coherent scatter is present). Furthermore, it is shown that these cross correlations provide a direct measure of magnitude of the coherent scatter. To that end, we will compare the results of the theory to examples from the set of observations used by JK10a.

Before discussing these data further, however, we first look at the theory for the cross-correlation function between neighboring range bins when both incoherent and coherent backscatter are occurring.

2. Theory

For a radar constant of unity, the net electric field at a location produced by spatially distributed scatterers can be expressed as

$$E(\mathbf{r}_1, t) = \sum_i a_i \exp[j(\omega_i t - 2\mathbf{k} \cdot \mathbf{r}_{1i})], \tag{1}$$

where a_i is the amplitude of the field scattered by the ith particle at $\mathbf{r}_i$ location from the observer, ω_i is its Doppler angular frequency, $\mathbf{k}$ is the wavenumber along the direction of propagation, and the factor of 2 accounts for a round trip. Similarly at range $\mathbf{r}_2 = \mathbf{r}_1 + \mathbf{l}$, where $\mathbf{l}$ is the separation between spatially independent (nonoverlapping) resolution volumes, we have

$$E(\mathbf{r}_2, t) = \sum_m a_m \exp[j(\omega_m t - 2\mathbf{k} \cdot \mathbf{r}_{2m})]. \tag{2}$$

It follows, then, that the cross correlation is

$$\begin{aligned}\langle E^*(\mathbf{r}_1, t)E(\mathbf{r}_2, t)\rangle &= \left\langle \sum_{i\neq m}\sum_{m\neq i} a_i a_m \exp[j(\omega_m - \omega_i)t] \times \exp[j2\mathbf{k}\cdot(\mathbf{l} - \Delta\mathbf{r}_{im})]\right\rangle \\ &= \left\langle \sum_{i\neq m}\sum_{m\neq i} a_i a_m \exp[j(\omega_m - \omega_i)t] \times \exp(j2\mathbf{k}\cdot\mathbf{l}) \times \exp(-j2\mathbf{k}\cdot\Delta\mathbf{r}_{im})\right\rangle,\end{aligned} \tag{3}$$

where $\langle\ \rangle$ represents the time average and $\mathbf{r}_m - \mathbf{r}_i = \mathbf{l} + \Delta\mathbf{r}_m - 0 - \Delta\mathbf{r}_i = \mathbf{l} - \Delta\mathbf{r}_{im}$ while the origin of $\mathbf{r}_1$ is set to zero for convenience. In JK10a [see discussion concerning Eqs. (A2)–(A4)] it is already established that, for Bragg scatter to occur, $\omega_i = \omega_m$ for some particles. Hence, after sufficient temporal averaging—<25–100 ms to allow particle reshuffling in rain and snow to decorrelate the signals to less than ~0.01 level using the normal assumption of an exponentially decaying (usually Gaussian) correlation function characterized by a $1/e$ decorrelation times of 5–20 ms (see JK10a, their Fig. 1; Jameson and Kostinski 2010b, their Fig. 1)—the $\omega_m \neq \omega_i$ term disappears $\langle E^*(\mathbf{r}_1, t)E(\mathbf{r}_2, t)\rangle$ and becomes

$$\langle E^*(\mathbf{r}_1)E(\mathbf{r}_2)\rangle = \left\langle \sum_{i\neq m}\sum_{m\neq i} a_i a_m \exp(j2\mathbf{k}\cdot\mathbf{l}) \times \exp(-j2\mathbf{k}\cdot\Delta\mathbf{r}_{im})\right\rangle = \exp(j2\mathbf{k}\cdot\mathbf{l})\left\langle \sum_{i\neq m}\sum_{m\neq i} a_i a_m \exp(-j2\mathbf{k}\cdot\Delta\mathbf{r}_{im})\right\rangle, \tag{4}$$

where $\Delta\mathbf{r}_{im}$ are the separations among the scatterers $i \neq m$ relative to a common center. Now the evaluation of the $\langle\ \rangle$ term in Eq. (4) follows that in JK10a, [appendix A, Eqs. (A5)–(A15)], since the particles in each volume simply obey different realizations of the same pair-correlation function. (The assumption of statistical homogeneity implies that both volumes posses the same pair-correlation function vis-à-vis the correlation–fluctuation theorem; Ornstein and Zernike 1914; Landau and Lifshitz 1980). It then follows that Eq. (4) becomes

$$\langle E^*(\mathbf{r}_1)E(\mathbf{r}_2)\rangle = \langle I_B\rangle \exp(j2\mathbf{k}\cdot\mathbf{l}) = \frac{\overline{N}^2\overline{a^2}2\pi}{Vk}\exp(j2\mathbf{k}\cdot\mathbf{l})\left\langle\int_0^\infty l\eta(l)\sin(-2kl)\,dl\right\rangle, \tag{5}$$

where I_B is the coherent back-scattered power, $\overline{N}$ is the mean number of particles in each sampling volume of size V, and l is the scalar distance in the direction of propagation. Clearly, then, the magnitude ρ_{12} is given by

$$\rho_{12} = |\langle E^*(\mathbf{r}_1)E(\mathbf{r}_2)\rangle| = \frac{\overline{N}^2\overline{a^2}2\pi}{Vk}\left|\left\langle\int_0^\infty l\eta(l)\sin(-2kl)\,dl\right\rangle\right| = \langle I_B\rangle \tag{6}$$

so that the fractional coherent contribution $\mathfrak{F}$ to the total power P is $\rho_{12} = |\langle E^*(\mathbf{r}_1)E(\mathbf{r}_2)\rangle|/\sqrt{Z_1Z_2}$, where Z represents the two radar reflectivity factors given by $\overline{Na^2}$. When $\eta = 0$ there is no structure (correlation) on any scale so that $\rho_{12} = 0$; that is, there is only incoherent scatter. Up to now the role of η has been neglected. For example, others have noted that $\rho_{12} = 0$ because neighboring range bins have no scatterers in common [e.g., Doviak and Zrnić 1993, p. 515, Eq. (C.8) with $\delta\tau_s = \tau$]. However, this conclusion is only valid for incoherent scattering because it is not the commonality among any of the scatterers in independent sample volumes, but rather the commonality of *structures in resonance with the radar wavelength* that produces $\rho_{12} \neq 0$ and, therefore, coherent scatter. Since by the assumption of statistical homogeneity $Z_1 = Z_2 = \overline{Na^2}$ and by letting $F_B = |\langle\int_0^\infty l\eta(l)\sin(-2kl)\,dl\rangle|$, $\mathfrak{F}$ can be expressed succinctly as

$$\mathfrak{F} = \frac{\overline{N}^2\overline{a^2}2\pi}{VkC\overline{a^2}\overline{N}}F_B = \frac{2\pi\overline{N}F_B}{VkC}, \tag{7}$$

where λ is the radar wavelength. Clearly, for the same F_B, V, and $\overline{N}$, $\mathfrak{F}$ increases with increasing λ. This λ comes from the inverse of wavenumber k. One possible physical explanation, then, for this wavelength dependence is that as λ decreases the number of waves in 2π radians increases so that coherent scatter sources may increasingly interfere with each other, leading to increasing cancellation of coherency as the wavelength decreases. In the limit of infinitesimal wavelengths, we would eventually have incoherent scatter. However, at this time this possible explanation can only be considered as speculation on our part. Furthermore, it must be remembered that Eq. (7) and hence the dependence on λ and V, only applies to statistically homogeneous conditions.

It is also worth noting that propagation phase shift has no effect on ρ_{12}. Hence, by making measurements of ρ_{12}, Z_1, and Z_2, one can calculate $\mathfrak{F}$ for various sample volume separations. However, if Bragg scatter is to be evaluated most directly, the separations of the sampling volumes should be relatively small so that the derivation in JK10a remains valid. Analyses of the data below show that, by the time the separation reaches ~300 m, ρ_{12} rapidly approaches the noise level. This is not too surprising given the apparently small spatial dimensions of the coherently scattering grids of particles (Jameson 2010c). That is, the separation must be small enough to satisfy the condition of sufficient statistical homogeneity so that the mean number of particles is the same in the two sampling volumes and so that the same pair correlation function exists in both volumes. Furthermore, it is highly desirable that the sample volumes be small so that there will be less noise and fewer other factors capable of producing decorrelation and, hence, of degrading estimates of $\mathfrak{F}$. Interestingly, the displacement between the sampling volumes by $\mathbf{l}$ becomes irrelevant as long as the precipitation remains statistically homogeneous.

In practice, when searching for coherent scatter, one wants to keep the origin fixed. Then after ρ_{12} is computed for that origin, it can be moved one bin and ρ_{12} can be recomputed so that any valid signals, if present, are not spatially or temporally averaged out of existence by the usual process of computing a cross-correlation function. We do this below using radar data from the National Science Foundation–Colorado State University–University of Chicago–Illinois State Water Survey (CSU–CHILL) Radar Facility at Greeley, Colorado.

3. Some observations

This radar has a 1.1° beamwidth. It operates at a frequency of 2.725 GHz corresponding to a nominal wavelength of 11.01 cm. Holding the antenna stationary, time series observations of the backscattered complex amplitudes (I, Q pairs) were collected 1024 times per second at vertical polarization. In the rain, observations were collected over 332 150-m range bins over a distance of about 3–53 km from the radar. The elevation angle was 1.82° so that the bottom of the main lobe of the beam was around 600 m above the surface at about 30-km range. These measurements are through weak convection containing a few convective cores. Likewise, observations were gathered in snow over 218 150-m range bins over a distance of about 3.30–36 km from the radar. The elevation angle was 2.54° so that the bottom of the main lobe of the beam was around 700 m above the surface at about a range of 20 km. Finally, I, Q measurements using a stationary antenna were collected at 30-m resolution in snow on 9 March 2009, at an elevation angle of 11.04° from

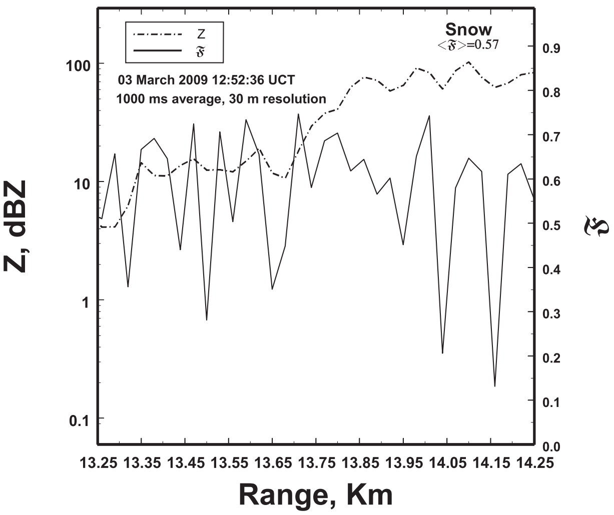

FIG. 1. Plots of Z and $\mathfrak{F}$ ($=\rho_{12}$) as functions of radar range over 1 km at 30-m resolution in snow. The value $\langle\mathfrak{F}\rangle$ is the mean of all $\mathfrak{F}$ plotted.

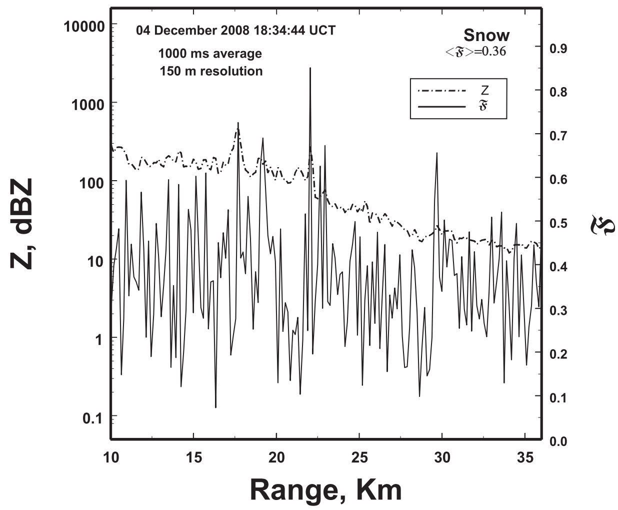

FIG. 2. As in Fig. 1, but for a different snow event and the resolution is only 150 m. The corresponding smaller value of $\langle\mathfrak{F}\rangle$ is likely due to enhanced decorrelation over a sample volume 20 times greater than in Fig. 1.

1.77 to 15.0 km. All evidence of any ground clutter contamination from any low-order side lobes disappeared by 11 km.

While none of these observations are as high resolution as we would like, as we shall see, they do provide an important beginning as well as the first direct measurements of coherent scatter from precipitation. We begin with the 30-m data after first describing the calculation procedure in greater detail.

a. *The procedure*

In these CSU-CHILL observations, the I, Q pairs are recorded at each range bin every 0.9766 ms over approximately a 1-min interval while the antenna remains fixed. The polarization was selected to be vertical. The data were first combined to be a complex amplitude. These were then stored in an array in which each column was a different range bin (usually a few hundred) while each row corresponded to each millisecond observation over one second, with time increasing with increasing row index. Beginning with the first range bin, that complex amplitude is complex conjugated and then multiplied with its next range neighbor, and that value is stored in a new array. This is again repeated for the next range bin and, subsequently, for all rows (times). The new array is then transposed and a cumulative sum is taken along each row, and each element of the array is then divided by the total elapsed time up to that array bin to yield a running temporal average for each range bin from 1 to 1000 samples. In the plots to follow, we only look at the 1000-sample mean values although it is possible to pick any time average from 0.001 to 1 s.

In what follows, calculations of the expected noise levels arising from chance for ρ_{12} based on characteristic times to decorrelation found in these data (20 and 5 ms in snow and rain, respectively) and on the assumption of correlated, incoherent scatter indicate that the vast majority of the plotted values of $\mathfrak{F}$ shown below are statistically meaningful. Specifically, the (mean, standard deviation) of the noise in snow and rain were calculated to be (0.144, 0.0726) and (0.0736, 0.0385), respectively. We also note that both the means and the standard deviations seem to vary as the square root of the number of independent samples.

b. *The results*

We begin with the 30-m resolution data plotted in Fig. 1. The mean $\mathfrak{F}$ over this 1-km radial is 0.57. This is smaller than the mean of 0.72 in snow at 150-m resolution in JK10a in the orthogonal direction for 57 radials and over 300 range bins in a different snow event. Reasons for this difference are discussed at the end of this paper. The most important point here, however, is that $\rho_{12} \neq 0$ anywhere.

This is also found when one looks at only one radial at 150-m resolution in the snow event considered in JK10a as shown in Fig. 2. The Z values are considerably larger in this example, while the sample volume is on the order of 20 times larger. Since this is only one radial, it is not surprising, then, that the mean is $\langle\mathfrak{F}\rangle = 0.36$ not only in part just by chance, but also because the larger sampling volume leads to enhanced decorrelation as discussed at the end of this work. The important point, however, is that once again $\rho_{12} \neq 0$ and $\mathfrak{F}$ is not zero anywhere.

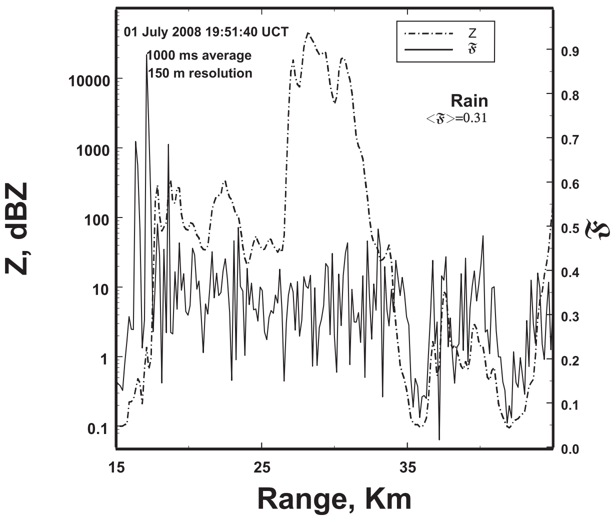

FIG. 3. As in Figs. 1 and 2, but for observations in rain.

Finally, we consider a radial in rain as discussed above and in JK10a (Fig. 3). The $\langle \mathfrak{F} \rangle$ is 0.31. In this case this is in better in reasonable agreement with the average value of 0.33 found in JK10a. The important point to note here is that just as in JK10a, $\mathfrak{F}$ appears to be smaller in the rain than in the snow. However, at this stage of investigation what is most important are not these limited quantitative comparisons but rather the fact that ρ_{12} and thus $\mathfrak{F}$ exist where classical incoherent scatter theory says there should be only noise.

4. Conclusions

While JK10a have already reported evidence for the presence of radar coherent backscatter, in that work it was argued that radar coherent scatter occurs in the direction perpendicular to the direction of wave propagation because of the presence of grids of enhanced particle concentrations with spatial periodicities in resonance with the radar wavelength. Here, however, and in support of the previous results, an approach is presented that provides a direct observation of radar coherent backscatter from precipitation in the direction of propagation. Specifically, by computing the cross-correlation function of nearest neighbors in range, it is shown that with temporal averaging, values greater than the noise level exist because of the persistence of relevant portions of the pair-correlation function across range bins, leading to radar coherent backscatter in the direction of propagation. Using this different approach, examples in rain and snow are consistent with values found in JK10a, suggesting that indeed radar coherent backscatter in precipitation is occurring and that it can be significant at times.

The similarity in the qualitative behavior of coherent scatter from rain and snow calculated using two different techniques—the $Z(f)$ approach in JK10a, where f is the frequency of the fluctuations in both Z and the amplitudes, and the ρ_{12} approach here—is gratifying. However, it is likely that the ρ_{12} approach systematically underestimates $\mathfrak{F}$, particularly for the larger sampling volumes. Consequently, the approach using $Z(f)$ likely provides the best overall quantitative estimates of $\mathfrak{F}$. In part this is because $Z(f)$ is not affected by decorrelation caused by noise (and other factors, such as the size of the sample volume). Furthermore, $Z(f)$ includes important sources of coherent scatter not necessarily detected in the cross-correlation functions between neighboring bins. In fact, the oscillations associated with f in one bin are very likely to be out of phase with those in neighboring bins so that the $Z(f)$ oscillations would actually decorrelate ρ_{12} rather than reinforce it, thus leading to overall smaller $\mathfrak{F}$ computed using ρ_{12} rather than $Z(f)$. Nevertheless, regardless of such quantitative concerns, the enduring point here is that coherent scatter can explain the existence of $\rho_{12} = \mathfrak{F}$ above the noise level while incoherent scatter cannot.

Finally, it should be mentioned that there is no way to "correct" for coherent scatter except to remove it from the observations since η (and hence $\mathfrak{F}_B$) are unknown [see Eqs. (6)–(7)]. This means that the best estimates of the "true" Z_t are given by $Z_t = (1 - \mathfrak{F})Z$. Moreover, when the radar antenna is scanning it is not possible to estimate $\mathfrak{F}$ because non-Rayleigh signal effects (Jameson and Kostinski 1996) make detection impossible (even though coherent scatter is still present). Hence, such corrections would require phase array antennas capable of dwelling at one azimuth before jumping to the next. Thus, for all current operational radars, no observation-to-observation correction is presently possible.

However, rather than attempting to correct for coherent scatter, a second approach is to minimize its relevance when one is trying to estimate rainfall, for example. In rain, the best way to do this is to use radar polarization measurements, specifically the combination of differential reflectivity Z_{DR} and differential phase K_{DP}, as discussed in Jameson (1994). Although coherent scatter will increase the variances in Z_{DR} and K_{DP}, it should not bias their mean values.

Regardless of such considerations, a different but important implication of the findings in this work is that observations in separate range bins cannot simply be combined, assuming statistical homogeneous conditions, and then treated as though they were statistically independent. Hence, the statistical reliability of estimates using techniques such as pulse compression to achieve high-spatial-resolution measurements, which can then be combined to yield estimates over larger domains, will likely be overestimated.

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