Phase signature for particle detection with digital in-line holography

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The spatial phase resulting from the digital reconstruction of an in-line hologram of a particle field is shown to yield a unique pattern that can be used for particle detection. This phase signature is present only when viewed along with the reference light. The existence of the phase pattern is verified computationally and confirmed in laboratory experiments with holograms of calibrated glass spheres. The phase signature provides an alternative to the widely used intensity method for particle detection. \bigcirc 2006 Optical Society of America

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Digital in-line holography, because of its simplicity in optical configuration and fast processing via digital recorder and computer, has generated much interest in three-dimensional particle image velocimetry^{1,2} and the study of three-dimensional particle spatial distributions.³ In these fields, the first step usually is the extraction of the spatial distribution of small particles in a three-dimensional volume. Typically, the forward-scattered light from a group of small particles, such as water droplets in clouds or seeding particles in a fluid, are initially recorded on a CCD camera by the in-line holography method, and then the forward-scattered light field is reconstructed numerically by filtering off the reference light (dc component of the hologram's Fourier transform) and neglecting the twin images of the particles. Most particle detection algorithms have been based on various features of the scattered field amplitude or intensity, whose spikes then suggest the presence of particles. $^{4-7}$ In practice, the spikes from the particles are distinguished from those due to noise by setting a threshold.

In this Letter we deviate from the traditional processing in two ways: (i) the reference light is kept (not filtered off), and (ii) rather than using amplitude or intensity we rely on the spatial phase. This spatial phase is that of the whole light field, not just of the scattered field, and is proposed here as an alternative for detecting particles.

It is important to note that this spatial-phase signature of the entire light field is not associated with (transparent) phase objects, the latter being the subject of a vast literature even within holography (e.g., see Vikram's text,⁸ especially Sec. 5.9, and references therein). Our phase signature is based on the opaque disk diffraction approach, whose adequacy, even for transparent objects, has been demonstrated in many holographic studies.^{9,10} Qualitatively, the focal length of the particle lens, say a water droplet, is of the order of its diameter *d* so that the subsequent spreading angle of this refracted light beyond the focal point is rather large $[\theta_r \sim \tan^{-1}(d/2d) \sim 1]$, whereas the spreading angle of the disk-diffracted light is considerably smaller ($\theta_d \sim \lambda/d \ll 1$). Hence a distant CCD array records mostly the latter (except, perhaps, in large-numerical-aperture applications such as microscopic imaging).

The typical digital in-line holographic configuration contains only collimating optics and a CCD that will record the interference intensity pattern between the collimated on-axis reference light and the forward-scattering light from the particles. The general principle for hologram recording and reconstruction can be found in, for example, Goodman's text,¹¹ and it has been adopted by many authors¹² for digital recording and reconstruction. For clarity, we will use



Fig. 1. (Color online) Phase signature reconstructed from a simulated hologram. (a) Spatial phase variation (within an axial plane through the center of the opaque disk) of the reconstructed field based on a simulated hologram. (b) Comparison of the axial phase reconstruction (solid line) with the 1/z decay predicted by Eq. (2) (dashed curve). It can be seen that the finite size effects of the pixels and CCD chip prevent the phase from following the 1/z dependence near the opaque disk, and the basic phase signature remains conspicuous and robust.

the continuous function representation. Given a linear recording of the in-line hologram with a normalized plane reference beam (amplitude normalized to 1), the light intensity $I(x, y, z_0)$ recorded on the CCD plane (x,y) located at z_0 can be described as¹¹ $I(x,y,z_0) = 1 + a_{z_0}(x,y) + a_{z_0}^*(x,y) + |a_{z_0}(x,y)|^2$, where *a* and *a*^{*} are the scattered wave and its conjugate, respectively. Within the Fresnel approximation, and treating the scattering from a small particle as that from an opaque disk,⁶ the scattered field $a_{z_0}(x,y)$ can be written as $a_{z_0}(x,y) = a_0(x,y) \otimes h_{z_0}(x,y)$, where $h_{z_0}(x,y) = (j\lambda z_0)^{-1} \exp[j(k/2z_0)(x^2+y^2)]$, $a_0(x,y)$ is the transmission function of the object at z=0 with $a_0(x,y) = -1$ within the disk and 0 outside the disk, \otimes denotes convolution operation, and λ and k denote the wavelength and corresponding wave number, respectively. The reconstructed complex whole field $E_z(x,y)$ at an interrogation plane of distance z, which is $E_z(x,y) = I(x,y,z_0) \otimes h_{z-z_0}(x,y)$, by neglecting the weak contribution from the twin image $a_{z_0}^*$ and scattered field intensity $|a_{z_0}|^2$ can be approximated as

$$E_z(x,y) \approx 1 + a_0(x,y) \otimes h_z(x,y). \tag{1}$$

Equation (1) will serve as the basis for interpreting the reconstruction results, and we note that the reference light will *not* be filtered out.

Given the form of $a_0(x,y)$, the on-axis field values of the reconstructed whole field $E_z(r=0)$ in cylindrical coordinates is

$$\begin{split} E_z(0) &\approx 1 - \frac{1}{j\lambda z} \int_0^{r_0} \int_0^{2\pi} \, \exp\!\left(j\frac{k}{2z}r^2\right) r \mathrm{d}\,\theta \mathrm{d}r \\ &\approx e^{j(l/z)} \equiv e^{j\phi(z)}, \end{split} \tag{2}$$

where $\phi(z) \sim z^{-1}$ is the axial phase, $l \equiv \pi r_0^2 / \lambda$ and r_0 is the opaque disk radius. Given that z is the axial distance from the particle, Eq. (2) describes a unique phase signature: a 1/z singularity near the particle, followed by a change of sign (phase flip) at the disk position and subsequent decay beyond the disk. A detailed computation based on Eq. (1) confirms this axial-phase signature and also shows that, laterally, there is a narrow phase wake behind and ahead of the particle confined to roughly $\pm r_0$ in the radial direction.

However, the finite sizes of the recording pixels and array of the CCD camera will limit the bandwidth of the digital in-line holographic system in recording reconstruction.¹² and Therefore the sharp axial-phase rise (or decay) predicted by Eq. (2) will be saturated because of the resolution limited by the pixel size and the array size (or CCD aperture size). To estimate the saturated value, we observe that the lateral phase variation within the region $(0, r_0)$ (the reference light field is not perturbed strongly outside at lateral distances $r > r_0$ so that $\phi \approx 0$ there) is well approximated by a linear function. Then, as required by the sampling theorem, the reconstructed axial phase value would not exceed $\sim 2\pi (r_0/2\rho)$, where ρ is the local spatial resolution. If the local spatial resolution is limited by the pixel size Δ , implying $\rho = \Delta$, then the axial phase is estimated to saturate near

$$\phi_1 = \pi(r_0/\Delta). \tag{3}$$

Similarly, if the local spatial resolution is limited by diffraction from the CCD aperture with size D, implying $\rho = \lambda(z_0 - z)/D$, and our concern is with the axial phase in the vicinity of the opaque disk $(z_0 - z \approx z_0)$, then the reconstructed axial phase is estimated to saturate near

$$\phi_2 = \pi (r_0 D / \lambda z_0). \tag{4}$$

Ultimately, the $\phi(z)$ saturation value is determined by the smaller of ϕ_1 and ϕ_2 , hereafter denoted min{ ϕ_1, ϕ_2 }, with the latter depending on the system parameter combination $\phi_2/\phi_1 = \Delta D/\lambda z_0$ (provided that the particle is resolved, i.e., $2r_0 \ge \rho$).

Figure 1 displays the particle phase signature reconstructed from a simulated hologram. It can be seen that the phase flip does occur at the opaque disk location as predicted by Eq. (2) and that the axial phase does saturate at values predicted by Eqs. (3) and (4). The parameters used in this simulation were chosen to match the laboratory data, presented in the next paragraph: $r_0=20 \ \mu m$, $z_0=77.4 \ mm$, Δ =4.65 μm , $\lambda=632.8 \ nm$, and $D=2.3 \ nm$. These parameters yield $\phi_1=13.5 \ rad$.



Fig. 2. (Color online) Phase signature reconstructed from experimental in-line holographic data. (a) Reconstructed spatial phase variation within an axial cross-section through the center of the glass sphere. (b) Reconstructed phase signature along the central axis through the particle. A close agreement of the spatial phase features with those of the simulation shown in Fig. 1 is observed. The reconstruction is based on the same array size and interrogation plane spacing as those for the simulation.

To disentangle fundamental optics from the numerical artifacts of phase unwrapping, we deliberately chose the CCD array size as 500×500 so that $\phi_2=3.0 \text{ rad } (<\pi)$. Hence $\min\{\phi_1, \phi_2\}$ is due to the diffraction from the finite CCD aperture. The spacing between the interrogation planes is $80 \ \mu\text{m}$. The maximum of the reconstructed phase in Fig. 1 is about 3.0 rad, which agrees with the predicted value for $\min\{\phi_1, \phi_2\}$.

To test the above theoretical and computational results, we have analyzed an in-line hologram of small particles recorded in the laboratory. Specifically, we have calculated the phase from the complex field obtained through digital reconstruction from the recorded hologram. The experimental system consists of a He–Ne laser, a spatial filter, a collimating lens for illuminating the particles, and a CCD camera for recording the hologram. The CCD contained 1024 \times 768, 4.65 μm square pixels, with 10 bit output. Glass spheres with a diameter of 40 μm , with National Institute of Standards and Technology traceable size calibration, were placed on a glass slide approximately 77 mm from the CCD.

Figure 2 shows the particle-phase signature reconstructed from the experimental in-line holographic data. It can be seen that the results match well with those of the simulation shown in Fig. 1. The experiment constitutes the proof of concept by demonstrating that the phase signature can be captured in practice and that the saturation value min{ ϕ_1, ϕ_2 } is consistent with the limitations imposed by finite size effects as given in Eqs. (3) and (4).

Of course, the phase signature can be used for particle detection only if it can be distinguished from the background noise, such as in the case of our experiment illustrated in Fig. 2. Can a much higher level of noise be tolerated? To quantify the level of noise that allows the particle-phase signature to be distinguished from the phase caused by noise, we consider the following conservative scenario: a phase ϕ within $(-\pi,\pi)$ reconstructed from a complex value $(\cos\theta)$ $+n_{\rm re}$) + $i(\sin \theta + n_{\rm im})$, where θ is the true phase value on $(-\pi, \pi)$, and $n_{\rm re}$ and $n_{\rm im}$ are the real and the imaginary parts of noise (relative to unity), respectively. Far from the particle, the reference light field is not strongly perturbed so that the true phase angle θ ≈ 0 . Thus the reconstructed phase angle can be approximated as $\phi_{\text{far}} \approx \tan^{-1}[n_{\text{im}}/(1+n_{\text{re}})]$. Near the particle, the reference light field can be strongly perturbed by the particle and the true phase angle θ $\approx \pm \pi$. The reconstructed phase angle can then be approximated as $\phi_{\text{near}} \approx \tan^{-1}[(1+n_{\text{im}})/n_{\text{re}}]$. The noise

level requirement \bar{n} can be roughly estimated by considering the worst case in which the larger phase value of $\phi_{\text{far}} \approx \tan^{-1}[\bar{n}/(1-\bar{n})]$ will reach the lowered phase value of $\phi_{\text{near}} \approx \tan^{-1}[(1-\bar{n})/\bar{n}]$. A simple calculation provides the maximum tolerable noise level of up to $\bar{n} \sim 50\%$ if the maximum reconstructed phase in the particle-phase signature is around or above π . Performing a simple Monte Carlo calculation shows that if $n_{\rm re}$ and $n_{\rm im}$ are independent and Gaussian distributed with zero means and sufficiently small variance; the reconstructed phase ϕ is similarly distributed except for a shifted mean of θ . This indicates that a simple phase threshold of, say, $P_{\text{threshold}}$ = $\pm 2\pi\sigma_n$, where σ_n denotes the standard deviation of the noise, may be used to distinguish the particleinduced phase from that of the background noise.

In this Letter, through theoretical analysis, numerical simulation, and experiment, we have demonstrated the feasibility of using the phase signature of particle fields reconstructed from digital in-line particle holograms for particle detection. This signature appears robust because it readily distinguishes the signal from the background noise. The approach provides an alternative to the widely used intensity method for particle detection and may suggest other possibilities for particle detection.

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