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Phase-modulated pupil for achromatic imaging of faint companions

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Abstract

Planets outside our solar system may harbour life but their faint images are lost in the bright diffraction haloes of their parent stars. Here we show with a simple example that it is possible to suppress the star haloes without substantially weakening the planet signal if starlight, entering a telescope pupil, is phase-modulated in a prescribed pattern. The modulation remains effective throughout the entire visible light band and for a wide range of view angles. Furthermore, "phase only" modulation over the pupil suggests an attractive possibility of using a single optical element (deformable mirror) for simultaneously removing diffraction haloes and correcting phase errors.

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The faint companion problem, that is, detection of a weak signal located near a much stronger source, arises in several fields (e.g., radar and sonar) but is, perhaps, best exemplified by the search for extra-solar planets. The expected planet-to-star brightness ratio ranges from about 10^{-7} to even lower than 10^{-10}, coupled with the angular separation possibly as small as 0.25 arcsec [1–3], make direct imaging difficult and still elusive. Any approach to overcoming this difficulty (low image contrast) must somehow rely on the weak diffraction pattern "symmetry breaking" by the planet, that is, the fact that planet's off-axial position displaces its peak intensity from the diffraction plane center.

Current approaches to the contrast improvement are motivated by the Fourier transform relationship between the pupil (aperture) and image (diffraction) intensity patterns [4]. This has led to several ingenious optical designs such as phase/amplitude masks and amplitude apodized pupils [3,5–9], but the required tolerance on wavelength purity is often tight. While coronagraphs (masks) are filters in the diffraction (focal) plane of the telescope, here we pose a different question: is there a phase modulation of the pupil itself, which can suppress starlight diffraction haloes without significantly weakening the planet signal, and do so for a wide range of wavelengths?

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Once asked, the question can be readily answered. While we found phase functions that yield deep reduction for several pupil shapes, including the circular one, the square pupil has emerged recently as a particularly attractive candidate [8,9]. In addition, the square pupil permits consideration of separable starlight signals, that is, light fields represented by $e^{-i\phi(x,y)} = e^{-i[\phi_1(x)+\phi_2(y)]} = e^{-i\phi_1(x)}e^{-i\phi_2(y)}$, and thereby, rendering the problem essentially one-dimensional. The following mathematical observation (given in one dimension for the sake of clarity) suggests that the desired phase patterns might exist when the search of the diffraction plane is conducted one quadrant at a time.

Let the phase over the pupil be $\phi(x)$ and the transmission function be $T(x)$, yielding the focal (diffraction) plane pattern

$$E(\eta) = \widetilde{T(x)e^{i\phi(x)}}, \tag{1}$$

where x is the coordinate in the pupil plane, η is the coordinate in the diffraction plane, $\tilde{}$ denotes the Fourier transform, and the transmission function $T(x)$ (rectangular unit pulse) is 1 for $|x| \leqslant D/2$, and zero otherwise (D being the width of the one-dimensional pupil). Given that the Fourier transform of $T(x)$ is $\mathrm{Sinc}(\eta) \equiv \sin(\pi\eta)/(\pi\eta)$ (which is real and even), Eq. (1) can also be written as

$$E(\eta) = \mathrm{Sinc}(\eta) \otimes \left\{\widetilde{\cos\left[\phi(x)\right]} + i\widetilde{\sin\left[\phi(x)\right]}\right\}, \tag{2}$$

where $\otimes$ denotes the convolution.

The crucial observation here is that when $\phi(x)$ is an odd function, the possibility arises of "destructive interference" between the $\cos[\phi(x)]$ and $\sin[\phi(x)]$ diffraction terms for an entire half of the Fourier domain. Indeed, in this case $\widetilde{\cos[\phi(x)]}$ is an even and real function but $i\widetilde{\sin[\phi(x)]}$ is odd and real. The light intensity on the diffraction plane is then a squared sum of two real functions (rather than sum of squares as in the case of even $\phi(x)$):

$$I(\eta) = \Big\{ \mathrm{Sinc}(\eta) \otimes \widetilde{\cos\left[\phi(x)\right]} + \mathrm{Sinc}(\eta) \otimes \mathrm{Im}\Big\{\widetilde{\sin\left[\phi(x)\right]}\Big\}\Big\}^2 \tag{3}$$

so that mutual cancellation becomes possible. Motivated by this argument, we conducted systematic numerical experiments and discovered that the desired phase patterns do indeed exist and deliver deep halo reduction (below 10^{-12}) throughout the entire quadrant of the focal plane. Not only do they exist, but also they are simple, smooth, and surprisingly stable to wavelength change.

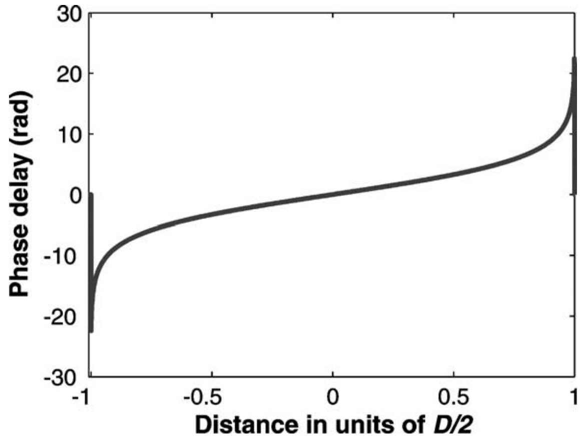

Fig. 1. 1D plot of the phase delay function, for the central wavelength λ_0, given by $\phi(x;\lambda) = \frac{\lambda_0}{\lambda} b \ln[f(x)]$ where $f(x) = \frac{(1+\varepsilon)+2x/D}{(1+\varepsilon)-2x/D}$, $b = 3$, $\varepsilon = 10^{-3}$.

Our numerical experiments, using the Gerchberg–Saxton iteration method [10], confirm that pupil phase functions $\phi(x)$ yielding deepest halo reduction are smooth and odd on the $(-\frac{D}{2}, \frac{D}{2})$ pupil interval, e.g., $\phi(x) \propto \tan(ax/D)$ where a is a numerical constant near π, and D is the side of a square pupil. In Figs. 1 and 2 we show another example of a wavelength-averaged diffraction (Fourier transform) pattern of $e^{-i\phi(x,y)}$ for a phase function of the form

$$\phi(x, y; \lambda) = \frac{\lambda_0}{\lambda} b \ln\left[f(x) \cdot f(y)\right], \tag{4}$$

where $f(x) = \frac{(1+\varepsilon)+2x/D}{(1+\varepsilon)-2x/D}$, b and ε are set to 3 and 10^{-3} respectively, λ_0 is the central wavelength. This function causes sufficiently deep reduction of the starlight diffraction halo throughout the entire quadrant of the image plane (Fig. 2) for the planet signal to be seen there. The pattern can then be rotated, and the process repeated. Note that most of the central region (main diffraction lobe) is preserved in order not to weaken the planet signal substantially.

The desired phase functions can, perhaps, be realized by mechanical, electrical, magnetic, and thermal methods but employing active high-density deformable mirrors may be the most promising approach because the phase error correction and the

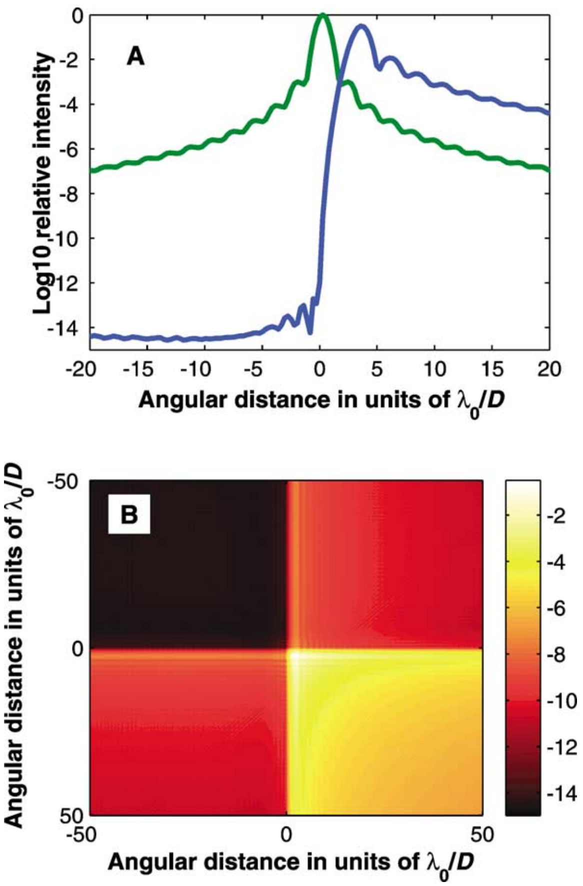

Fig. 2. Annulling of the star diffraction haloes by a two-dimensional extension $\phi(x, y; \lambda) = \phi(x; \lambda) + \phi(y; \lambda)$ of the phase modulation function in Fig. 1. (A) Blue curve is the diffraction pattern (discrete Fourier transform based on 12 800 samples of $e^{-i\phi(x)}$) of the phase-modulated starlight, plotted vs. the angular position along a diagonal of the image plane for the first $20\lambda_0/D$, normalized to the peak intensity of the modulation-free diffraction pattern (the green curve). Even for the broad band (the bandwidth of $0.6\lambda_0$), covering the entire visible range, the phase modulation still yields a deep contrast of below 10^{-9} at $3.5\lambda_0/D$ from the peak and remaining below 10^{-12} from about $4\lambda_0/D$. (B) displays the broader view of the annulling effect over the entire second quadrant of the focal plane. A uniform contrast of 10^{-14} over this quadrant has been reached, sufficient for direct imaging of extra-solar planets with a moderate telescope.

halo removal can be implemented simultaneously by the same deformable mirror. The performance of this method will depend mainly on the position accuracy and stability of the deformable mirrors and precision of the square pupil, whose fabrication is quite feasible. Instead of rotating the optical elements or system, sequential searching of the whole diffraction plane can be achieved by rearranging the actuator stroke values. Given modern high-density deformable mirrors with position accuracy and stability of 1 angstrom/100 hour and 0.01 angstrom/hour, respectively, in controlling the actuators [11], the phase delay shape can be controlled to less than 10^{-3} rad/100 hour in the visible light. Our simulations demonstrate that a continuous deformable mirror driven by only a 21×21 actuator array can already provide sufficient contrast of below 10^{-9} in one quadrant of the diffraction plane. This is based on the simple “Gaussian hill” response of a deformable mirror [12] where the Gaussian declines to 10% of the peak value at the center of an adjacent actuator. Furthermore, simulations of a simple segmented mirror driven by 201×201 “piston only” and by 101×101 “piston and tilt” actuators yield the contrast of below 10^{-9}.

Many of the past approaches have been plagued by either the high sensitivity to wavelength, by the limited range of view angles, and also by the complexity of the implementation, so the potential simplicity and broad spectral range of the phase-modulated pupil method are appealing. As can be seen from Fig. 2, deep reduction in starlight diffraction halo levels is not significantly affected by the wavelength change throughout the entire visible range. Such flexibility allows immediate spectroscopic study, once a planet is detected.

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