

Some Laplace Transform Pairs

	$f(t)$	$F(s)$
1.	1	1/s
2.	t	1/s ²
3.	e^{-at}	1/(s+a)
4.	te^{-at}	1/(s+a) ²
5.	$\frac{t^{n-1}}{(n-1)!} e^{-at}$	(s+a) ⁻ⁿ
6.	sin(ωt)	$\omega/(s^2+\omega^2)$
7.	cos(ωt)	s/(s ² + ω^2)
8.	$e^{-at} \sin(\omega t)$	$\omega/[(s+a)^2+\omega^2]$
9.	$e^{-at} \cos(\omega t)$	(s+a)/[(s+a) ² + ω^2]
10.	$\sqrt{c^2+d^2} e^{-at} \cos\left(\omega t - \tan^{-1}\left(\frac{d}{c}\right)\right)$	$\frac{c(s+a)+d\omega}{(s+a)^2+\omega^2}$
11.	sinh(ωt)	$\omega/(s^2-\omega^2)$
12.	cosh(ωt)	s/(s ² - ω^2)
Special Transforms		
13.	$\frac{df}{dt}$	s F(s) - f(0 ⁺)
14.	$\int_0^t f(\tau) d\tau$	F(s)/s
15.	$c_1 f_1(t) + c_2 f_2(t)$	$c_1 F_1(s) + c_2 F_2(s)$
16.	$\int f_1(\tau) f_2(t-\tau) d\tau$	F ₁ (s) F ₂ (s)
17.	$\delta(t-T)$ <small>(delta function, T > 0)</small>	e^{-sT}
18.	$\frac{d^n}{dt^n}(\delta(t)) = \delta^{(n)}(t)$ <small>(n-th derivative of delta function)</small>	s ⁿ
19.	$\delta^{(-n)}(t)$ <small>(n-th integral of delta function)</small>	s ⁻ⁿ

Comments on Transforms.

- A) The transform variable, s , is complex. The Fourier transform is the special case of this more general transform where s is restricted to be pure imaginary.
- B) In the table above, a , c , and d are considered real constants, though many of the transforms are still valid if they are complex or pure imaginary.
- C) 1 to 4 are special cases of 5.
- D) 1, 3, and 6 to 9 are special cases of 10.
- E) 6 and 7 are related to 11 and 12. If you replace ω with $i\omega$ in one pair, you generate the other.
- F) 15 is a statement of the linearity of the transform.
- G) 14 is a special case of 16 (where one of the functions is constant).
- H) 16 is known as the “convolution theorem.”
- I) 18 is particularly useful if you have an equation with a delta-function in it and need to take a derivative. This occurs for some idealized models of real systems (*e.g.* In E&M, current through a thin wire is often modeled using a delta function for the current density.).