Small Argument Exercises

In each case, find an appropriate approximate expression correct to the lowest order term which includes the "small" variable.

Example:

$$\frac{b}{(a + x\sin\varphi)^2}\ ;\ |x| << |a|$$

Here x is small compared to a, so an appropriate small parameter is x/a, but note also that $|\sin\varphi| \le 1$ so that can be included too. Then using the binomial expansion

$$\frac{b}{(a + x\sin\varphi)^2} = \frac{b}{a^2}\frac{1}{(1+\frac{x}{a}\sin\varphi)^2} = \frac{b}{a^2}(1+\frac{x}{a}\sin\varphi)^{-2} = \frac{b}{a^2}\left(1+(-2)(\frac{x}{a}\sin\varphi) + \frac{(-2)(-3)}{2!}(\frac{x}{a}\sin\varphi)^2 + \ldots\right)$$

so to the lowest (non-zero) power of x, we have the approximation

$$\frac{b}{(a + x\sin\varphi)^2} \approx \frac{b}{a^2}\left(1 - \frac{2x}{a}\sin\varphi\right)$$

1. $(a - 3x)^9 \quad ; \quad |x| << |a|$

2. $\dfrac{1}{\sqrt{R^2 + 2Rr\cos\theta + r^2}} \quad ; \quad r > 0,\ R > 0,\ r << R$

3. $A\cos^2(kr)\cos(5kr)\ ; \quad |kr| << 1$

Use the expansion of cosine: $\cos(\epsilon) \approx 1 - \frac{1}{2}\epsilon^2$, $\epsilon << 1$, or see equations 12.68 to 12.73 in Mathematical Handbook of Formulas and Tables, 2nd Ed, by Spiegel and Liu. The binomial coefficients used there are defined on page 6.