PH3110 Vector Exercises

You should be able to do these without a calculator (except for inverse trig functions and possibly a square root) or reference book.

1. For vectors **A** = 2 **i** + 3 **j** - 4 **k** and **B** = **2** i **-4** j + **3** k compute:

(A) **A**+**B**
(B) **A**-**B**
(C) **A**·**B**
(D) **A**×**B**
(E) the angle between **A** and **B**
(F) the components of **A** in spherical coordinates.
(G) the angle between **B** and the y axis.
(H) for the tensor ***D*** shown, compute **A**·***D***·**B** .
(I) compute $\boldsymbol{D}^T\boldsymbol{D}$.

$$D = \frac{1}{5}\begin{pmatrix} 3 & 4 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

2. Suppose a tensor ***R*** corresponds to a rotation in three dimensions. With **C** any vector (in three dimensions), use a geometric argument to explain why the following will be true.

$$(\boldsymbol{R}\mathbf{C})\cdot(\boldsymbol{R}\mathbf{C}) = \mathbf{C}\cdot\mathbf{C}$$

3. Show that for any three vectors, **A**, **B**, and **C**, that

(A) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{C}\times\mathbf{B})\times\mathbf{A} = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$

(See if you can do part A more elegantly than by simply writing out components and applying brute force)

(B) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0$

(Hint: instead of writing out components, try using the identities in part A for part B)