

Name _____

Final Exam - PH3110

Fall 2003

DUE: Thursday, 18 Dec., 4:30 pm. You may use your text book, class notes, and a math table but do not consult with anyone other than your instructor. Remember to show solutions, not just answers!

1. Given the three vectors

$$\mathbf{A} = 3\hat{x} - 2\hat{y}$$

$$\mathbf{B} = 2\hat{y} + 5\hat{z}$$

$$\mathbf{C} = (r = \sqrt{2}, \theta = 90^\circ, \phi = 45^\circ)$$

where \mathbf{A} and \mathbf{B} are in rectangular coordinates, and \mathbf{C} is given in spherical coordinates (see figure 1.12.2 of your text, if needed), compute the following:

(A) $\mathbf{A} \cdot \mathbf{B}$

(B) $\mathbf{A} \times \mathbf{B}$

(C) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$

2. A particle of mass, m , moves in one dimension, x , and is subject to a total force

$F = -Cmv^\alpha$, where C ($C > 0$) and α are constants and v is speed. The minus sign indicates that the force opposes the motion. The particle starts at $t = 0$, $x = 0$ with speed $v = v_0$ in the positive x -direction, and comes to a stop after a time t_0 at a position x_0 . What are C and α (in terms of m , v_0 , t_0 , and x_0) ?

3. A particle of mass, m , moves in a one dimensional potential, $U(x) = \alpha x^4 - \beta x^2$, where α and β are positive constants.

(A) What is the force, $F(x)$, associated with this potential?

(B) Find the position(s) of stable equilibrium, and

(C) the frequency (or frequencies) for small oscillations about the positions found in (B).

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4. In three dimensions, a particle is subject to a conservative force, $F(\mathbf{r})$,

$$F(\mathbf{r}) = 3y^2x\hat{\mathbf{x}} + cx^2y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$$

where c is a constant.

(A) What is c ?

(B) What is the potential energy, $U(\mathbf{r})$, associated with this force?

5. For each of the following, briefly (but completely) describe what it is, where it shows up, etc.

- a) Coriolis force.
- b) Quality factor, Q .
- c) Terminal velocity.
- d) Apsidal Angle.
- e) Critical damping.
- f) Space-time interval.

6. A very fast moving car traveling straight North at constant velocity passes a policeman. The policeman sees that the car is moving at $0.200c$. By an amazing coincidence, both the clock in the car and the policeman's clock read exactly $t = 0$ at the time they pass. Being concerned, but safety conscious, the policeman drives slowly (< 55 mph) North and after traveling 20.0 km finds that the car had crashed into a tree. Assuming the clock in the fast moving car stopped at the first moment of impact (and that it is still readable after the collision), what time will the policeman read off the broken (but previously very precise) clock in the car?

7. A particle moves in a central force following a path described in plane polar coordinates by

$$\theta = \frac{c}{r^2}, \text{ where } c \text{ is a positive constant and } \theta > 0. \text{ Show that } \theta \text{ increases exponentially with}$$

time and hence r decreases exponentially with time. You may find it useful to express your answer using $l = |\mathbf{v} \times \mathbf{r}|$, a constant of the motion.

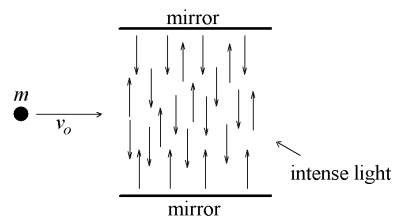
8. The radius of the Moon's orbit about the Earth is increasing at a rate of about 3.5 m/century due to tidal interactions between the Moon and Earth.

(A) Is the Moon's angular momentum about the Earth increasing, decreasing or staying the same?

(B) Is the orbital energy of the Moon about the Earth increasing, decreasing, or staying the same?

(For both of these you must, of course, give good reasons for your answers).

9. Consider a spherical black object (it absorbs all light which hits it) of mass m moving in the x -direction with initial speed v_0 . The object enters a region between two mirrors where there is very intense light traveling in the $\pm y$ -direction. Explain why the light causes the object to slow down.



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