Gradient and Curl in different coordinate systems

In each of the following, *f* is a scalar function of position, **A** is a vector function of position. The subscripts denote the components of the vectors in the appropriate coordinate system.

(A) Rectilinear Coordinates (*x*, *y*, *z*)

$$(\nabla f)_x = \frac{\partial f}{\partial x} \quad ; \quad (\nabla f)_y = \frac{\partial f}{\partial y} \quad ; \quad (\nabla f)_z = \frac{\partial f}{\partial z}$$

$$(\nabla \times \mathbf{A})_x = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \quad ; \quad (\nabla \times \mathbf{A})_y = \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \quad ; \quad (\nabla \times \mathbf{A})_z = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

(B) Cylindrical Coordinates (r, ϕ, z)

$$(\nabla f)_r = \frac{\partial f}{\partial r} \quad ; \quad (\nabla f)_\phi = \frac{1}{r}\frac{\partial f}{\partial \phi} \quad ; \quad (\nabla f)_z = \frac{\partial f}{\partial z}$$

$$(\nabla \times \mathbf{A})_r = \frac{1}{r}\frac{\partial A_z}{d\phi} - \frac{\partial A_\phi}{\partial z} \quad ; \quad (\nabla \times \mathbf{A})_\phi = \frac{\partial A_r}{dz} - \frac{\partial A_z}{\partial r} \quad ; \quad (\nabla \times \mathbf{A})_z = \frac{1}{r}\frac{\partial (rA_\phi)}{dr} - \frac{\partial A_r}{\partial \phi}$$

(C) Spherical Coordinates (r, θ, φ)

$$(\nabla f)_r = \frac{\partial f}{\partial r} \quad ; \quad (\nabla f)_\theta = \frac{1}{r}\frac{\partial f}{\partial \theta} \quad ; \quad (\nabla f)_\varphi = \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}$$

$$(\nabla \times \mathbf{A})_r = \frac{1}{r\sin\theta}\frac{\partial (A_\varphi \sin\theta)}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \varphi}$$

$$(\nabla \times \mathbf{A})_\theta = \frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial (rA_\varphi)}{\partial r}$$

$$(\nabla \times \mathbf{A})_\varphi = \frac{1}{r}\frac{\partial (rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}$$