Hyperbolic Trig. Functions

(part of the "exponential family")

$$\sinh(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\cosh(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\operatorname{sech}(x) = 1/\cosh(x)$$

$$\operatorname{csch}(x) = 1/\sinh(x)$$

$$\operatorname{ctnh}(x) = 1/\tanh(x)$$

Some identities:

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh x + \sinh x = e^{x}$$

$$\cosh x - \sinh x = e^{-x}$$

(Note the similarity to sines and cosines, which are also part of the exponential family, but with some significant sign changes).

Some derivatives:

$$\frac{d(\sinh x)}{dx} = \cosh x \quad ; \quad \frac{d(\cosh x)}{dx} = \sinh x \quad ; \quad \frac{d(\tanh x)}{dx} = \operatorname{sech}^2 x$$

$$\frac{d(\operatorname{sech} x)}{dx} = -\operatorname{sech} x \tanh x \; ; \quad \frac{d(\operatorname{csch} x)}{dx} = -\operatorname{csch} x \operatorname{ctnh} x \; ; \quad \frac{d(\operatorname{ctnh} x)}{dx} = -\operatorname{csch}^2 x$$