PH3110 Homework - Sp. Relativity 2

1. A policeman uses a radar gun and the doppler shift to measure your speed. The gun emits microwaves (a form of non-visible light) at a particular frequency, f_0, which then reflect off your car and are subsequently received by the policeman.

a) If you are moving at a speed v towards the policeman, what microwave frequency will you observe?

b) In your car's reference frame, the reflected microwaves will have the frequency computed in part a. What change in frequency will the policeman see for the reflected microwaves compared to the emitted microwaves? (Hint: if there is not a change, then the police would not find radar guns very useful.)

c) If f_0 = 2 GHz and v = 55 mph, what is the *change* in frequency in Hz?

d) Show in general that in the limit $v/c \ll 1$, the frequency *shift* is proportional to $f_0\, v/c$.

2. In a laboratory, an electron and a proton are each accelerated from rest through an electric potential difference of V (usually measured in volts). Assume V may be large enough so that relativistic effects should be considered. ($m_e c^2$ = 0.511 MeV, $m_p c^2$ = 938 MeV).

a) What is the ratio of the speed of the electron to the speed of the proton (both in the laboratory reference frame) as a function of the potential V?

b) Show that in the limit where V is small, you get the classical result $v_e / v_p = \sqrt{m_p / m_e}$.
(Hint: this is a check on the correctness of your answer to part a).

3. Show that any particle which has zero rest mass but non-zero energy must be traveling at speed c. (Hint: write v as a function of m for fixed $E \neq 0$, then take the limit $m \to 0$). A photon (a "particle of light") is one such massless particle.

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4. In class it was shown that for relativistic motion in 1-dimension along x with constant proper acceleration, α, and starting from rest at $t = 0$ one gets

$$\frac{dx}{dt} = \sqrt{\frac{\alpha^2 t^2}{1 + \alpha^2 t^2 / c^2}}$$

and from this one can derive the equation of motion for 1-d constant proper acceleration (along x and starting from rest at the origin):

$$x = \frac{c^2}{\alpha}\left[\sqrt{1 + \frac{\alpha^2 t^2}{c^2}} - 1\right]$$

(a) Show that in the limit of small speeds (i.e. $\alpha t \ll c$) this gives the classical result. (In class we showed that for small speeds, $\alpha \approx a$, where a is our usual everyday acceleration.)

(b) Show that based on the equations above, a photon (which travels at a speed c) *must* have an infinite proper acceleration. (Hint: be careful with your algebra – one way to solve this is to assume that α is finite and show that this leads to a contradiction except in the limit $\alpha \rightarrow \infty$). Basically this result tells you that you cannot stop a photon.